Journal of Economic Dynamics & Control 153 (2023) 104697

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Intergenerational transfers: Public education and pensions with endogenous fertility[☆]

Monisankar Bishnu[a,b,c,*], Shresth Garg[d], Tishara Garg[e], Tridip Ray[a]

[a] *Economics and Planning Unit, Indian Statistical Institute, Delhi, India*
[b] *Centre for Applied Macroeconomic Analysis (CAMA), ANU, Australia*
[c] *CEPAR, UNSW, Australia*
[d] *Department of Economics, Harvard University, United States*
[e] *Department of Economics, MIT, United States*

ARTICLE INFO

Article history:
Received 12 December 2022
Revised 22 June 2023
Accepted 23 June 2023
Available online 28 June 2023

Keywords:
Intergenerational Transfers
Education - Pension Package
Fertility
Welfare State

ABSTRACT

We consider an overlapping generations economy, where parents are altruistic towards their children, and children provide old-age support to parents. We show that when the education loan market is imperfect, an education subsidy targeted towards achieving the complete-market level of education distorts fertility decisions. However, augmenting the education policy with pension support in the old-age can restore both education and fertility to complete-market level. This highlights that an Education-Pension package is more potent than perceived by the existing literature – it not only replaces the missing credit market but also corrects for fertility distortions. Our results also hold when state intervention in education is justified due to human-capital externality, instead of credit market frictions.

1. Introduction

Existing research has shown that credit markets to borrow funds for education are imperfect everywhere.[1] This market failure prevents the investment in education from reaching its optimal level, providing rationale for state intervention. While the agents receive government support for education when they are young, in many countries they also receive income support from the state as pensions after they retire. In fact, education subsidies and Pay-As-You-Go (henceforth PAYG) pensions are the two largest welfare programs undertaken by governments around the world.[2]

☆ The authors are grateful to the Editor and two anonymous referees for their excellent comments and suggestions. We would also like to thank Emmanuel Farhi, James Poterba, Morten Ravn, and Iván Werning for their helpful comments.

* Corresponding author.
E-mail addresses: mbishnu@isid.ac.in (M. Bishnu), garg@g.harvard.edu (S. Garg), tishara@mit.edu (T. Garg), tridip@isid.ac.in (T. Ray).

[1] Economists have long emphasized imperfections in the education loan market (Friedman and Kuznets (1945); Friedman (1955; 1962) Nerlove (1975); Chapman (2006)). See Becker (1962) for an overview of why capital market imperfection tends to be explicitly stressed in the context of under-investment in education.

[2] Government spending accounts for 91% of funds at primary, secondary and post-secondary levels and 70% at the tertiary level in OECD countries (see OECD (2017a)). Public education spending in the United States alone accounts for 4.2% of GDP and 11.8% of total public spending (see OECD (2017a)). As for pensions, the OECD (2017b) report on public pensions states that ‘Public pensions are often the largest single item of social expenditure, accounting for 18% of total government spending on average in 2013’. The old-age support expenditure, as a percentage of GDP in OECD countries, stands at 8.6%.

https://doi.org/10.1016/j.jedc.2023.104697
0165-1889/

The education support and pensions are typically funded through taxes on the middle-aged, but benefits the young and the old. An education subsidy program, which involves transfers from the middle-aged to the young, might not be palatable to the initial working population who foot the bill for the program but do not benefit from it. This makes an education subsidy challenging to sustain. Pensions get around this implementation hurdle by compensating the generation bearing the burden of the education program. This has been documented in an important contribution by Boldrin and Montes (2005) where they show that when the education loan market is imperfect, an appropriately designed Education-Pension (henceforth EP) package can achieve the complete-market level of education.[3] The idea of interlinking education and pensions this way is not new and goes back to at least Hammond (1975) and Pogue and Sgontz (1977). Becker and Murphy (1988) argue that parents might not invest optimally in children's education if such an investment competes with their retirement provision. A welfare state can overcome this deficiency by taxing parents and financing public education. Poterba (1996; 1998) discuss this link in the context of the US economy shedding light on the issue of sustaining the forward intergenerational good (education) as the population ages and demands more backward intergenerational good (pension). Rangel (2003) also emphasizes this interlinkage by arguing that backward intergenerational goods play a crucial role in sustaining forward intergenerational goods; without them investment is inefficiently low.[4] Many papers have investigated the interlinkage in different contexts, including, recently, Andersen and Bhattacharya (2017) and Bishnu et al. (2021) which study interdependence of intergenerational transfer instruments focusing more on the possibility of eventual phase out of PAYG pensions.

The impact of intergenerational instruments on fertility has been largely ignored in the literature mentioned above. Fertility is typically viewed as being exogenous to education and pension policies. In this study, we consider an economy with endogenous fertility where parents are altruistic towards their children, and care about the quantity and quality (education) of their children. In return, children provide old-age support, through an intergenerational contract. Thus, education and pension policy may have a first order impact on fertility choices. We find that while government support for education, motivated by market distortions, restores education choices to their complete-market level, ends up distorting fertility choices. Augmenting the education policy with pension support restores both education and fertility to the complete-market level. Therefore, the instruments that a welfare state has at its disposal are potentially more powerful than previously realized.

In a related work, Conde-Ruiz et al. (2010) consider a model with endogenous fertility and provide a characterization of efficient allocation as the equilibrium of a decentralized market economy with mandated intergenerational transfers. Their analysis also documents that when credit markets for human capital is incomplete, linking pension and education subsidy is required, to restore efficiency. Our model differs in significant ways. In our setup, unlike theirs, parents are altruistic with a quality-quantity trade-off and crucially, rely on children for old age support through an implicit inter-generational contract. Altruism and old-age support are important factors when considering fertility decisions. We continue to find EP package necessary, even allowing for altruism and old-age support. We provide new results that EP package continues to be needed when the market failure arises due to human capital externality instead of credit market frictions.[5]

Our paper also contributes to many strands of literature. In a related paper, Schoonbroodt and Tertilt (2014) show that if parents do not have enough property rights over children's income, equilibrium fertility will be inefficiently low and, unlike in the exogenous fertility case, a PAYG system cannot be used to implement the efficient allocations.[6] Like them, we also model inefficiency in the fertility choice, although arising because of forward-looking transfers instead of a lack of ownership over children's income. In fact, in our setup, parents have full claim over their children's income. Interestingly, in our setup, pensions emerge as the instrument that helps to restore efficiency. Therefore, we see our paper as complementary to theirs and further, from the above mentioned perspective, our result in fact strengthens the findings such as Hammond (1975), Pogue and Sgontz (1977), Becker and Murphy (1988), and Boldrin and Montes (2005), for an entirely different reason.

Most economic models of fertility choice are built on the notion of a quality-quantity trade-off between the number of children and education per child.[7] If fertility and education are indeed joint decisions, government policy aimed at altering the cost of education might have unintended consequences for fertility. In this paper, we show that fertility overshoots its optimal level when the government intervenes solely on the education front.[8] We then show that, not only can an EP package correct market failure and help achieve the optimal level of education, but it can also help a welfare state

[3] *Complete-market* level is the one which prevails when the loan market for education is perfect.

[4] In this way, pensions also help to overcome the political economy constraints in introducing a forward generational transfer, especially in an environment of the growing influence of old in the society. Sustainability and related issues of forward and backward intergenerational transfers in a political economy setting have also been discussed in the literature, see for example, Bishnu and Wang (2017), Gonzalez-Eiras and Niepelt (2012), Lancia and Russo (2016) among others.

[5] A recent body of literature has emerged to address efficiency-related issues in the context of endogenous fertility. Conde-Ruiz et al. (2010) extends the Pareto efficiency criteria to an efficiency concept called Millian efficiency, both in a static and dynamic sense. The static criteria require equality of returns from the capital market and from investments in children, while the dynamic efficiency criteria state that the rate of return to physical capital should exceed the rate of return to investments in children, as measured by the ratio of wages to the cost of children. In our paper, we focus on the notion of dynamic efficiency in the standard sense, which refers to economies where the market rate of return on capital exceeds the rate of population growth. Although our model structure differs, our efficiency concept aligns with the concepts of RC and CRC efficiency in Michel and Wigniolle (2007), as well as the $\mathcal{A}$-efficiency concept in Golosov et al. (2007), which are fundamental requirements in Conde-Ruiz et al. (2010). The analysis of dynamic efficiency under stochastic production has been examined in Zilcha (1991).

[6] They abstract away from issues of inefficiency in human capital.

[7] See, for example, Becker and Lewis (1973), Cigno (1986), Becker et al. (1990), De la Croix and Doepke (2003).

[8] In this paper, our benchmark is the equilibrium with no market failure. Hence, we use 'optimal' and 'complete-market' level interchangeably.

correct distortions in the fertility choice. Thus, our contribution lies in showing that a welfare state equipped with an EP package can do a lot more than just acting as a replacement for the imperfect education loan market: it can help achieve the optimal level of fertility in the economy. Interestingly, our results continue to go through when we introduce a human capital externality as an alternative rationale for state intervention in education. This suggests that our results are robust to how one models market failure.[9]

The economic approach to fertility emphasizes the role played by the cost of rearing and educating children. Barro and Becker (1988) note that there is a sizeable bequest to children through investment in education, a form of bequest that is far more common than the transfer of assets. If the cost of education does indeed play a role in the fertility choice of parents, government policy that affects the opportunity cost of education can have first order effects on fertility.[10]

Children are typically considered as a consumption good, with emotional gratification and altruism being the primary motivation behind begetting them. Barro and Becker (1988) use a utility function for parents which, in addition to their consumption, depends directly on the utility of each child and the number of children. Other examples of papers that model fertility this way are Doepke (2004) and Cigno (2006). Another way to model fertility as a consumption good is where parents derive utility from the 'quality' and 'quantity' of children (see, for example, Becker (1960); Becker and Lewis (1973); Cigno (1986)). However, there is a strand of literature that models children as investment goods instead. Here the motivation for having children is that they provide income support in old age. In Boldrin and Jones (2002) and Boldrin et al. (2015), for example, altruism runs from children to parents. Parents' old-age consumption enters the utility of their children, providing them with a reason to support the parents in old age. Parents base their fertility choice on this anticipated transfer. In papers like Ehrlich and Lui (1991) and Cigno (1993), parents and children have an implicit contract wherein parents invest in the education of children in return for old age support.

Building on these models of endogenous fertility, a few papers have tried to theoretically establish the empirically observed negative relationship between fertility and social security.[11] Barro and Becker (1988) show that, when the economy is dynamically efficient, a permanent increase in social security benefit leads to a temporary fall in fertility. Both Cigno (1993) and Ehrlich and Lui (1998) establish a negative relationship arising from the fact that the state provided pensions reduce parents' reliance on the family arrangement for old age support. Boldrin et al. (2015) establish a negative relationship quantitatively, albeit coming entirely from general equilibrium effects.

We consider an overlapping generations economy where fertility choice is endogenous and parents procreate out of both self-interest and altruism. Parents have an implicit contract with their children wherein they invest in their human capital in return for old-age support. They have access to the loan market for education where imperfections raise the effective cost of borrowing.[12] We show that the government intervention in education can restore education to its complete-market level. However, this intervention fails to correct fertility, which ends up overshooting the complete-market level. There exists a negative relationship between fertility and pensions under the PAYG regime. The government can exploit this negative relation and restore fertility to its complete-market level. Thus, a policy of only intervening in education loan markets is not sufficient; an EP package can replicate the complete-market level of both education and fertility. This leads us to conclude that an EP package is much more capable than existing studies have led us to believe–it not only acts as a substitute for the missing credit market but also helps attain the optimal level of fertility in the economy when the education loan market is not perfect.

Let us briefly explain the mechanism that is at work. Both fertility and educational investment are sub-optimal due to the high cost of financing education under *laissez-faire*. When the government intervenes with an education subsidy, fertility increases as the private cost of education falls. However, the cost of borrowing remains uncorrected so that borrowing is less than the complete-market level. Reduced borrowing decreases the loan repayment burden, increasing effective income. Since children yield a non-material benefit, they are a net pecuniary cost to parents on the margin. The increase in effective income allows the parents to raise more children, increasing fertility over and above that under complete market.

With fertility above its complete-market level, the state can use pension support to restore fertility to its optimum. Even though pensions transfer income from middle to old age, they affect consumption smoothing behavior. Agents borrow against this increased old-age income, transferring income back to middle age. The rate of return through government mediated PAYG pensions is the growth rate of the population. When population growth rate is lower than the cost of borrowing, as is the case in a dynamically efficient economy, the net effect of pensions is to reduce old-age income. The reduced old-

[9] Some other justifications for government involvement in education include a) production of social capital (Putnam et al. (1993); Durlauf and Fafchamps (2005)), b) creating a civic society of knowledgeable voters (Dee (2004)), and c) consumption externalities (Bishnu (2013)).

[10] There are many papers which note the impact of government policy on fertility. Cigno (1986) shows that if fertility is endogenous, government taxes and subsidies that affect the cost of children have an impact on the fertility decision and other market behaviors of the parents. Doepke (2004) shows that differential government policy regarding education plays a role in explaining the cross-country differences in fertility decline to a large extent. De la Croix and Doepke (2003) analyze public education policies in a set up where fertility and education are joint decisions and show that fertility dynamics are different in different education regimes, with implications for inequality and growth in those regimes.

[11] There is a substantial empirical literature confirming the negative relationship between fertility and social security. Hohm (1975) analyses data from 67 countries and shows that social security programs have a measurable negative effect on fertility, in a magnitude comparable to traditional correlates of fertility like infant mortality, education, and per-capita income. Country-specific analyses are carried out by Cigno and Rosati (1990) for Italy, Cigno and Rosati (1996) for Germany, Italy, UK and USA, and Cigno et al. (2003) for Germany.

[12] Showing market imperfection through differential borrowing rates is standard, say for example, as in Galor and Zeira (1993).

age income allows agents to support fewer children, leading to a negative relationship between pensions and fertility. The state can exploit this negative relationship to bring fertility back to its complete-market level.[13]

Our results are independent of the type of market failure motivating a state intervention in education. There is a line of literature which emphasizes human capital externality as one of the primary reasons for government intervention in education. We show that our results hold even when that is the case; how one models market failure is not critical to the results. While an education subsidy helps to correct externalities, reduces the effective cost of borrowing and improves the effective income of parents, it makes fertility overshoot its optimal level. A PAYG pension decreases fertility by lowering the income of parents in a dynamically efficient economy. In fact, we find that the channels through which an EP package works are broadly similar in both the models. Both credit market imperfection and externalities affect fertility via their impact on the cost of raising children and the borrowing pattern of the agents, with the two channels working in opposite directions.

Finally, our model is equally relevant in the case of developing economies which are in the midst of demographic transition, while still enjoying healthy population growth. Old age support is one of the most important motives for having children in developing economies (Leibenstein (1957); 1975). These are also the economies that are typically characterized by under-developed capital markets and a lack of institutionalized social security or old-age income insurance mechanisms.[14] In fact, Nugent (1985) identifies the absence of a well-organized capital market and public old age programs as an important condition under which the old age security motive for having children is likely to be important. He also argues that these conditions are likely to prevail in the rural areas of developing countries. Private transfers are a major component of household income and expenditure in developing countries and much of the transfer income flows from children to parents.[15] There is some evidence from developing countries that the initial introduction of social security crowds out such private transfers.[16] Becker et al. (2016) note that government-financed pension benefits provide elderly parents with incomes, reducing their need to rely on children for support. We argue, in that case, that pensions can alter the trade-offs involved in the fertility decision, suggesting that the introduction of public pensions can help hasten the demographic transition in developing economies.

The rest of the paper is organized as follows. Section 2 outlines our model. Section 3 describes the equilibrium that emerges under *laissez-faire*. Section 4 describes the equilibrium under a policy where the government intervenes only in education. In Section 5, we show the existence of the Education-Pension package that can replicate the complete-market levels of education and fertility. We provide an example of the recommended EP package for the specific case of CRRA utility function in Section 6. We discuss time consistency of the intra-family contracts in Section 7. We present the case with a human capital externality in Section 8, and Section 9 concludes. All the proofs are presented in the Appendix.

2. Model

We consider an overlapping generations economy where agents live for three periods. They are young in the first period, middle-aged in the second and old in the third. Time is discrete and indexed by $t = 0, 1, 2, ., \infty$.

In our notation, we identify a generation by the period of their middle-age. That is, we call an agent as belonging to the generation t if she is middle-aged in period t. In their young age, agents acquire human capital funded by their parents. The level of human capital H is realized in adulthood and is assumed to be a strictly increasing and concave function of the investment in education e. The human capital also has a non-acquired component, which we call the raw human capital and denote by $\bar{H}$. As is standard, we We assume that the human capital production function is concave and takes the form $H(e) = h(\bar{H} + e)^{\alpha}$ with $h > 0$ and $0 < \alpha < 1$. This means that e_{t-1} amount of investment on generation t in period $t-1$ generates human capital $H_t = h(\bar{H} + e_{t-1})^{\alpha}$ in period t. For now, we assume that this human capital production function is devoid of any externalities from the parental level of education or the average human capital in the economy. In Section 8 we discuss the case where externality from the average human capital in the economy is present. The factor prices are assumed to be exogenous.

We abstract away from the labor-leisure decision of the agents and assume that the agents are endowed with one unit of labor which they supply inelastically in their middle age, earning a total income of $a_t + H(e_{t-1})$.[17] Here a_t denotes the non-

[13] Even when fertility is assumed to be exogenous, an EP package is recommended whenever credit market is imperfect, as shown in Boldrin and Montes (2005). This recommendation is from the view of long run sustainability. We also recommend an EP package but for a different reason. Endogenizing fertility in our model reveals that an EP package is warranted to correct a distortion in the level of fertility caused by a subsidy in education.

[14] Pension Coverage is found to be positively related to per-capita income, ranging from over 90% for high-income OECD countries to less than 20% for South Asia and Sub Saharan Africa. The pattern of pension expenditure has been found to be correlated with the demographic structure as well. OECD countries with the highest old-age dependency ratio spend more than 8% of their GDP on pensions (social security is the highest public overhead in the US, for example) while South Asian and Sub Saharan African countries spend only 1-1.5%. (See, for example, Pallares-Miralles et al. (2012), Palacios and Sluchynsky (2006), OECD (2017b)). Also Becker et al. (2016) reports that "about 60 percent of the worlds elderly population receives no old-age pension. This fraction is over 80 percent in Sub-Saharan Africa and about 70 percent in Asia and the Pacific. To maintain a decent standard of living, many elderly parents either have to save enough or rely on their children."

[15] See, for example, Rempel and Lobdell (1978), Knowles and Anker (1981), Butz and Stan (1982), Ravallion and Dearden (1988), Cox and Jimenez (1990), Amuedo-Dorantes and Pozo (2006) for country-specific studies.

[16] See Cox and Jimenez (1992), for example.

[17] There is no uncertainty in our setup. Conesa and Krueger (1999) deals with idiosyncratic uncertainty in an heterogeneous agents setup when PAYG pension is present.

labor income.[18] Without loss of generality, we assume the rental rate of human capital to be unity. An agent of generation t chooses the number of children n_t and the amount to be invested in their education e_t. Raising children also entails a fixed rearing cost of γ per child. Parents have access to the education loan market from where they can borrow to fund a part (b_t) of the investment in children's education.

In their old age, agents repay any education loan that they took for their children's education.[19] However, they receive income support from their children.[20] Besides the material old-age support, children provide direct emotional gratification to parents in their old age. This direct utility gain depends on both the number as well as the quality of children. We assume that the utility function of a generation t agent takes the form

$$\mathcal{U}_t \equiv u(c_t^m) + \beta u(c_{t+1}^o) + \delta u(n_t^{\phi} H_{t+1}^{\psi}), \tag{1}$$

where c_t^m and c_{t+1}^o are the consumption levels in the middle age and the old age respectively.We assume that parents derive utility from the quantity and quality of children. Another frequently used modelling assumption follows Barro and Becker (1988, 1989), where parents directly care about the utility of their children. As the children care about their children, every generation ends up caring about all the subsequent generations. This introduces interdependence across time, making the problem intractable. The parameter β is the intertemporal discount factor: utility from consumption in the old age is discounted at the rate $\beta \in (0, 1)$ in the middle age. The last term captures the direct utility benefit from having children and is increasing and concave in the number of children and their human capital. This formulation of utility is in line with, for example, De la Croix and Doepke (2003, 2004). The parameter $\delta \in (0, 1)$ captures the degree of filial altruism. The function $u(.)$ is assumed to be strictly increasing, strictly concave and following Inada conditions, that is, $u'(.) > 0$, $u''(.) < 0$ with $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$. We assume $\phi > \psi$, that is, the marginal benefit of having a child is more valuable than her performance in later life (human capital).[21]

2.1. Direct old-age income support from children

We model old-age income support in a fashion similar to Ehrlich and Lui (1991). Parents enter into an *implicit* contract with the children. As per the terms of the contract, parents receive a proportion d_{t+1} of their children's labor earnings. This makes the per child contribution to the parent $d_{t+1}H_{t+1}$. In return, parents invest in the human capital of their children. Note that the old-age support received by the parents is increasing in the human capital investment they make in their children.

The proportion d_{t+1} is endogenously determined although not necessarily by direct negotiation between the parties to the contract. Parents can unilaterally choose an amount of compensation that is optimal for the children. In Section 7 we show that, in the presence of appropriate trigger strategies, such an arrangement is self-enforced and neither of the parties has an incentive to deviate from the contract.

2.2. Incomplete credit market

Parents have the option of funding a part of children's education expenses through the education loan market. They can borrow a proportion $b_t \in [0, 1]$ of the education expenses in their middle-age and repay the loan when they are old, out of the income support they receive.

We assume a small open-economy, and in particular, that the market rate of return R is determined in the international capital market. Imperfections characterize the credit market driving the cost of borrowing ρ above the market rate of return on savings R. These additional costs of borrowing can be justified in a setup where the lenders incur monitoring costs to ensure that the borrowers do not run away (for example, as in Galor and Zeira (1993)). Other market failures can also push the effective interest rate above the market interest rate (see, for example, Stiglitz and Weiss (1981)). This problem is more severe in the market for education loans as, unlike physical capital, human capital is inalienable and cannot be mortgaged (see, for example, Friedman (1962), Nerlove (1975) and Chapman (2006)).

[18] Our results are independent of the existence of raw human capital or the non-labor income, and would hold true even when $\bar{H} = 0$ and $a_t = 0$.

[19] Borrowing on behalf of children for their education is quite frequent and sometimes constitutes a substantial portion of household debt even in developed countries (for the U.S., Looney and Lee (2018)). There is also a prominent literature on education policy that is founded on the assumption that parents decide on children's education (see, for example, De Fraja (2002)). Note that, in our setup, since we have only three periods, any one-period loan taken by parents in their middle age has to be repaid when they are old.

[20] Another way this arrangement could be interpreted (without any change to the model) is that parents pass on their debt obligation to the children to recover the cost incurred on them. Children pay back the education loan taken by parents out of their own income. Hence, the old-age income support takes the form of both direct income transfer as well as payment of parents' loan obligation. Schoonbroodt and Tertilt (2014) note that in pre-industrialized US and England, parents had legal access to their offsprings' labor income. This alternative interpretation is particularly relevant in the context of developing countries where parents still exercise considerable control over their children's income and the laws protecting the earnings of children are either weak or non-existent.

[21] This may be interpreted as capturing some of the early age emotional benefit before the human capital has been realized.

3. *Laissez-faire* equilibrium

Parents' optimization problem involves a two-stage maximization procedure. In the first stage, they decide the quality and quantity of children, taking the old age transfer proportion (d_{t+1}) as given. The *first stage* optimization exercise yields an investment rule, that is, the parental investment in children's education as a function of d_{t+1}. In the second stage, parents decide the transfer proportion that is *optimal* for the children. They choose d_{t+1} so as to maximize the children's utility ($\mathcal{U}_{t+1}$) subject to their own investment rule.[22]

In the private equilibrium with no government, the first stage optimization entails deciding on a) the number of children (n_t), b) investment in education (e_t), and c) the proportion of borrowing (b_t) in their middle age, taking the old-age transfer proportion d_{t+1} as given. The problem of an agent of generation t is to maximize (1) subject to

$$\begin{aligned}
&a_t + H_t(1-d_t) = c_t^m + (e_t(1-b_t)+\gamma)n_t,\\
&H_{t+1}d_{t+1}n_t = c_{t+1}^o + e_t b_t n_t \rho,\\
&H_{t+1} = h(\bar{H}+e_t)^{\alpha},\\
&e_t, n_t, b_t \geq 0.
\end{aligned} \tag{2}$$

The agent maximizes her utility subject to the budget constraints in the middle and the old age and the human capital production function. In addition to the non-labor income a_t, the agent earns H_t in the middle age, of which she gives a fraction d_t to the parents as old-age income support. The total investment in the education of children is $e_t n_t$, of which she funds b_t proportion via the education loan market. Besides, parents bear a child-rearing cost of γn_t. Non-labor income and the earnings net of old-age transfer fund the child-rearing cost, the remaining part of educational investment ($e_t(1-b_t)n_t$) and own middle-age consumption (c_t^m). In her old age, the agent receives a total support of $H_{t+1}d_{t+1}n_t$ from all of her children which she uses to repay the education loan and fund old-age consumption.

As shown in Appendix A.1, the first order optimality conditions for problem (2), with respect to e_t, n_t, b_t respectively, assuming interior solution, reduce to[23]

$$\beta u'(c_{t+1}^o)\left(\rho n_t - d_{t+1}n_t\alpha\frac{H_{t+1}}{\bar{H}+e_t}\right) = \delta u'(n_t^{\phi}H_{t+1}^{\psi})n_t^{\phi}\psi\alpha\frac{H_{t+1}^{\psi}}{\bar{H}+e_t}, \tag{3}$$

$$\beta u'(c_{t+1}^o)\left(\rho(e_t+\gamma) - d_{t+1}H_{t+1}\right) = \delta u'(n_t^{\phi}H_{t+1}^{\psi})n_t^{\phi-1}\phi H_{t+1}^{\psi}, \tag{4}$$

$$u'(c_t^m) = \beta\rho u'(c_{t+1}^o). \tag{5}$$

The left-hand side of equation (3) captures the net marginal *material* cost of investing an extra unit in the education per child (in present value utility terms). An additional unit of educational investment increases the total cost of education by n_t, repayment of which would cost ρn_t in the old age. However, given the old-age transfer rate d_{t+1}, the additional unit will increase the human capital of children, which would, in turn, increase the income support from children coming in the old age. Thus, the net material cost increases by the difference of the two, which has to be multiplied by $\beta u'(c_{t+1}^o)$ to transform it to the present value utility terms. At the optimum, the net marginal *material* cost of an additional unit of educational investment is equal to the marginal *non-material* benefit from it, which is captured by the right-hand side of the equation.

Similarly, the left-hand side of the equation (4) captures the net marginal *material* cost of having an additional child. An additional child will cost $e_t+\gamma$ while it will yield an additional old-age income support of $H_{t+1}d_{t+1}$. The net marginal *material* cost of an additional child is equated to the *non-material* marginal benefit from it.

Equation (5) is the standard Euler equation capturing the trade-off in the borrowing decision. Borrowing an additional unit allows the agent to enjoy higher consumption in the middle-age yielding them a utility gain of $u'(c_t^m)$. However, it increases the loan repayment burden in the old age by ρ units with a utility loss of $\rho u'(c_{t+1}^o)$ which the agent discounts at the rate β.

Using the first order conditions (3), (4) and (5), we show in Appendix A.1 that the educational investment choice of the parents is given by an implicit function of the transfer proportion d_{t+1},

$$\rho(\gamma Z - \bar{H}) + d_{t+1}H_{t+1}(\alpha - Z) - \rho e_t(1-Z) = 0, \tag{6}$$

[22] The two-step maximization procedure is isomorphic to a Stackelberg game where children choose d taking parents' strategies $s^p(d)$ as given. It is consistent with the modeling assumptions – the chosen actions are the result of a dynamic game between parents and children where each of them is acting in self-interest. We show that the solution to the two-step procedure is, in fact, rationalized as the subgame perfect equilibrium of this dynamic game, supported with appropriate trigger strategies. Note that, the way the problem has been set up, each player plays the static stage-game twice – the parents and children play the stage game in period i followed by children and their children playing the same game in period $i+1$ (and so on).

[23] The problem is concave given the assumption that quantity and quality of children are complements in the utility function of the agent, that is, we assume that the marginal non-material benefit of having more children increases in the education level of the children.

where $Z \equiv \frac{\alpha\psi}{\phi} < \alpha < 1$ is the weight on the investment in education of children relative to that on their number in the altruism function of the parents. We see that the investment in education is increasing in the old-age transfer proportion contracted upon.[24]

In the *second stage*, parents decide the transfer proportion that is optimal for the children, taking their own investment rule as given. They choose the proportion that maximizes their children's utility ($\mathcal{U}_{t+1}$), subject to the equation (6). Assuming interior solution and imposing the envelope condition, the first order optimality condition for d_{t+1} becomes

$$\frac{\partial \mathcal{U}_{t+1}}{\partial d_{t+1}} + \frac{\partial \mathcal{U}_{t+1}}{\partial H_{t+1}}\frac{\partial H_{t+1}}{\partial d_{t+1}} = 0.$$

Solving the equation we get (see Appendix A.2)

$$d_{t+1} = \alpha\left(1 - \frac{(\gamma - \bar{H})Z}{(e^{IM} + \bar{H})(1 - Z)}\right), \tag{7}$$

where e^{IM} denotes the *laissez-faire* level of education under imperfect market and is given by

$$e^{IM} = \left(\frac{\alpha h(\alpha - Z)}{\rho(1 - Z)}\right)^{\left(\frac{1}{1-\alpha}\right)} - \bar{H}. \tag{8}$$

Investment in education decreases with an increase in ρ[25], with the complete-market level of education (e^{CM}) defined as that at $\rho = R$. Since $\rho > R$, we have $e^{IM} < e^{CM}$.[26]

4. Government intervention in education

The investment in education under imperfect market is less than the complete-market level. This is due to the higher (effective) cost of borrowing in the presence of market imperfections which distorts the incentives to invest in education. Under-investment in education creates room for government intervention.

We first explore the scenario where the government intervenes solely on the education front, by giving a constant subsidy s per unit of education and child care. That is, the government underwrites a fraction s of the total education and child care expenditure $\big(e_t(1 - b_t) + \gamma\big)n_t$[27]. This subsidy is financed by levying a lump-sum tax τ_t on the working population. The government balances its budget period by period. The budget balancing equation of the government becomes

$$\tau_t = s_t\big(e_t(1 - b_t) + \gamma\big)n_t. \tag{9}$$

With the government subsidy in place, the agent maximizes (1) subject to

$$\begin{aligned}
&a_t + H_t(1 - d_t) = c_t^m + \big(e_t(1 - b_t) + \gamma\big)(1 - s_t)n_t + \tau_t,\\
&H_{t+1}d_{t+1}n_t = c_{t+1}^o + e_t(1 - s_t)b_t n_t \rho,\\
&H_{t+1} = h(\bar{H} + e_t)^\alpha,\\
&e_t, n_t, b_t \geq 0.
\end{aligned} \tag{10}$$

As shown in Appendix B.1, the first order conditions for problem (10), with respect to n_t, e_t and b_t respectively, assuming interior solution, are

$$\beta u'(c_{t+1}^o)\left(\rho(1 - s_t)n_t - d_{t+1}n_t\alpha\frac{H_{t+1}}{\bar{H} + e_t}\right) = \delta u'(n_t^\phi H_{t+1}^\psi)n_t^\phi\psi\alpha\frac{H_{t+1}^\psi}{\bar{H} + e_t}, \tag{11}$$

$$\beta u'(c_{t+1}^o)\Big(\rho(1 - s_t)(e_t + \gamma) - d_{t+1}H_{t+1}\Big) = \delta u'(n_t^\phi H_{t+1}^\psi)n_t^{\phi-1}\phi H_{t+1}^\psi, \tag{12}$$

[24] $\frac{\partial e_t}{\partial d_{t+1}} = \frac{H_{t+1}}{\rho(\frac{1-Z}{\alpha-Z}) - d_{t+1}\alpha\frac{H_{t+1}}{\bar{H}+e_t}} > 0$. The inequality follows from the sign of the denominator which is positive because investing in education has a net positive material cost on the margin, as can be seen from equation (3).

[25] In this particular set-up, we assume that the parameter values are such that agents borrow in equilibrium. In particular we assume that the (implied) rate of return on children *on the no savings corner* is sufficiently large – $d(1+n) > 1 - (e+\gamma)\frac{n}{H} + \frac{a-\tau}{H}$, and the rate of return on savings is sufficiently small – $1 < \beta R < \beta\rho < \frac{u'(a-\tau+H-(e+\gamma)n)}{u'(Hdn)}$. This leads education to be sub-optimal in presence of credit market imperfections. Note, however, that the set-up is expositional and the general argument in the paper is independent of how one models education market imperfections (also discussed in Section 8 with the help of an example).

[26] For further discussion, we define complete-market level of human capital $H^{CM} = H(e^{CM})$, and complete-market level of old-age transfer proportion $d^{CM} = \alpha\left(1 - \frac{(\gamma-\bar{H})Z}{(e^{CM}+\bar{H})(1-Z)}\right)$. Note that the transfer proportion under imperfect market is less than that under complete market.

[27] The form of the subsidy transfer is not crucial for the subsequent analysis. Specifically, the analysis would stand if we assume government gives a lump-sum subsidy instead.

$$u'(c_t^m) = \beta \rho u'(c_{t+1}^o). \tag{13}$$

In Appendix B.2, we show that the investment in education in presence of education subsidy s_t is

$$e^E(s_t) = \left(\frac{\alpha h(\alpha - Z)}{\rho(1-s_t)(1-Z)} \right)^{(\frac{1}{1-\alpha})} - \bar{H}. \tag{14}$$

By giving an education subsidy s_t, the government achieves $e^E(s_t)$ level of investment in education. Comparing (8) and (14) we see that the government can replicate the complete-market level of investment in education by setting $s_t = s^*$ such that

$$(1-s^*)\rho = R. \tag{15}$$

Thus, the government corrects distortions in education by bringing back the effective cost of borrowing borne by the parents to the complete market level. Given that the government finances the subsidy by taxing the middle-aged agents who are also the beneficiary of the subsidy, the net transfer from the government is zero. However, since the agents act atomistically, not internalizing the effect of their education and fertility decisions on the total tax burden of the economy, it changes their incentives, and they end up investing more and begetting more children than they would have under *laissez-faire*.

4.1. Impact on fertility

The policy intervention discussed above affects not only the education decision but also the fertility choice. The education subsidy alters the effective cost of raising children. With the subsidy in place, the total education cost otherwise borne solely by the parent is now shared between her and the government. Even though the government finances its share by taxing the parent herself, she does not internalize this tax burden while taking her fertility decision. The only cost that the agent takes into account while deciding on the number of children is the one borne by her directly. Since this private cost of educating children goes down, fertility increases above its *laissez-faire* level.

Without any government intervention, both education and fertility are below the complete-market level. The subsidy is engineered to correct the education choice but ends up impacting fertility as a by-product. As is summarized in the following proposition, the altered fertility choice ends up overshooting the complete-market level.

Proposition 1. *Fertility under laissez-faire is less than the complete-market level. An education-subsidy-only package which corrects for distortion in the education level fails to correct for all distortions in fertility. Fertility under the package overshoots the complete-market level.*

Proof. See Appendix C. □

Children yield a non-pecuniary benefit in the form of filial altruism $\left(\delta u(n_t^{\phi} H_{t+1}^{\psi})\right)$ and a pecuniary benefit in the form of old age support $\left(d_{t+1}H_{t+1}n_t\right)$. The only cost associated with the children is the pecuniary cost of providing them with education and child care $\left(n_t(e_t+\gamma)\rho\right)$.[28] In an equilibrium with interior solution for fertility, the non-pecuniary marginal benefit from children is equated to the *net* pecuniary marginal cost (see equation (4) reproduced below for convenience). Thus, children are a net material cost for the parents.

$$\underbrace{\beta u'\left(c_{t+1}^o\right)\left(\rho(e_t+\gamma) - d_{t+1}H_{t+1}\right)}_{\textit{Net}\ \text{pecuniary marginal cost}} = \underbrace{\delta\left(u'\left(n_t^{\phi}H_{t+1}^{\psi}\right)\phi n_t^{\phi-1}H_{t+1}^{\psi}\right)}_{\text{Non-pecuniary marginal benefit}}$$

Imperfection in the education loan market distorts the fertility choice via two channels. It increases the cost of educating children, making children more expensive. Moreover, since market imperfection reduces the level of education investment, the expected old-age income support from children falls making them less attractive as an investment good. Besides, less educated children yield less altruism benefit on the margin. We call this the *effective cost channel*. This channel works in the direction of reducing fertility.

The second channel works via distortions in the borrowing decision and the resulting income effect. Market imperfection increases the cost of borrowing, due to which borrowing under *laissez-faire* is less than the complete-market level. Reduced borrowing decreases the loan repayment burden, increasing the effective income. Since children are a net pecuniary cost, this increase allows parents to have more children. We call this the *borrowing channel*. This channel works in the direction of increasing fertility.

When the loan market is imperfect, the effective cost channel dominates the borrowing channel. This leads to fertility being sub-optimally low under *laissez-faire*. When the government intervenes with a subsidy engineered to achieve the

[28] Even though the costs of education and child care are borne in the middle age while the old age support is received in the old age, the Euler equation (5) allows the net cost to be interpreted in terms of old-age consumption.

complete-market level of education, it corrects for the *net* per unit pecuniary cost of educating and rearing children $\big((1-s_t)\rho(e_t+\gamma)-d_{t+1}H_{t+1}\big)$, as $(1-s_t)\rho = R$ by design. Thus, the intervention shuts down the dominant *effective cost channel*. But the *borrowing channel* remains operative, as the true cost of borrowing (ρ) stays uncorrected. This makes fertility (sub-optimally) overshoot its complete-market level.

5. Government with education-pension package

Government intervention in education increases fertility beyond the complete-market level. Since pensions enter the budget constraint of the parents and one of the reasons parents have children is the old-age support, the government can influence fertility by intervening in the same with pension support.[29]

We now explore the scenario where the government provides pension support to the old, in addition to the education subsidy discussed above. Like the education subsidy, government finances this old-age income support by levying a lump-sum tax on the middle-aged. The budget balance equation of the government now becomes

$$\tau_t = s_t(e_t(1-b_t)+\gamma)n_t + p_t, \tag{16}$$

where p_t is the pension tax per middle-age working agent in period t. The total pension support that the generation t agent gets in his old age in period $t+1$ is given by $P_{t+1} = n_t p_{t+1}$.

The problem of the agent with an Education-Pension package is to maximize (1), subject to

$$\begin{aligned} &a_t + H_t(1-d_t) = c_t^m + (e_t(1-b_t)+\gamma)(1-s_t)n_t + \tau_t,\\ &H_{t+1}d_{t+1}n_t + P_{t+1} = c_{t+1}^o + e_t(1-s_t)b_t n_t \rho,\\ &H_{t+1} = h(\bar{H}+e_t)^\alpha,\\ &e_t, n_t, b_t \geq 0. \end{aligned} \tag{17}$$

In the following proposition, we show that the government can utilize the pension arm to restore fertility to its complete-market level.[30]

Proposition 2. *When rental rate of capital exceeds the population growth rate as in standard dynamically efficient economies, the government can replicate the complete-market levels of both education and fertility by using an Education-Pension (EP) package.*

Proof. See Appendix D. □

Pensions affect consumption directly through taxes and transfers (*direct income effect channel*) and indirectly through their impact on the borrowing decision of the agents (*borrowing channel*). The direct effect entails a transfer of income from middle age to old age. As discussed in Section 4, an increase in old-age income makes children less expensive on the margin, leading to an increase in fertility.

However, pensions affect fertility indirectly via its impact on borrowing. Since pensions transfer income from middle to old age, agents borrow more to smoothen their consumption. This adjustment in borrowing increases middle age consumption while the old-age income falls due to increased loan repayment. As old-age income falls, the net material cost of having children increases on the margin, leading to a decrease in fertility.

The return from government-mediated pension transfer is n, while the rate of return on transfer via borrowing is R. We show in a Lemma in Appendix D that when R is greater than n as in a dynamically efficient economy, the impact via borrowing dominates the direct income effect, leading to a reduction in fertility. The lemma claims that there exists a negative relationship between fertility and PAYG pensions in a dynamically efficient economy. This finding is also related to the literature that questions utility specification of fertility through the lens of negative income-fertility relation and the assumption that children are normal good (see for example, Córdoba and Ripoll (2016)). In Jones and Tertilt (2008), Manuelli and Seshadri (2009) and Daruich and Kozlowski (2020), fertility seems to decrease with the level of income and these empirical observations are therefore compatible with this Lemma in Appendix D.

As is shown in Appendix D, this negative pension-fertility relationship can be exploited by the government to bring fertility back to the complete-market level. We show the existence of an EP package which replicates the complete-market level of both education and fertility. The optimal EP package corrects for the distortionary income effect via borrowing, in addition to correcting for the effective cost of educating children. An education-only package distorts borrowing and consequently income, leading to fertility overshooting the complete-market level. The optimal pensions, via their impact on the borrowing in the opposite direction, cancel out the income distortion, making fertility come back to its complete-market level.

We already show the existence of a positive level of pensions that, along with optimal subsidy, replicate the complete-market level of education and fertility, for a general utility function in Appendix D. In the next section, we provide an example of the optimal EP package for the particular case of CRRA utility function.

[29] See, for example, Barro and Becker (1988), Ehrlich and Lui (1998) and Boldrin et al. (2015).

[30] In our model fertility falls with old-age income as parents need to rely less on children for old-age support. The same relationship holds for pensions, and is consistent with empirical evidence.

6. An example

Imperfection in the education loan market increases the cost of education and keeps the investment in education below the optimum. The state can subsidize education to bring it back to its complete-market level. However, in doing so, it fails to correct distortions in the fertility choice of the agents. We have shown that an education-only package leads fertility to overshoot its complete-market level. Education arm alone corrects education but not fertility. However, if the education arm is appropriately combined with the pension arm, distortions in both education and fertility choice of agents can be corrected.

We show in Appendix E that when the utility function is CRRA, that is, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, the Education-Pension package that replicates both the complete-market level of education and fertility is given by

$$s^* = 1 - \frac{R}{\rho}, \qquad p^* = \frac{(K-1)\big(Hdn + R[a + H(1-d) - n(e+\gamma)]\big)}{K(R-n)}, \tag{18}$$

where n, d, e and H are the complete-market levels of fertility, old-age transfer proportion, education and human capital respectively, and

$$K \equiv \left(\frac{\rho}{R}\right)^{\frac{1}{\sigma}-1}\left(\frac{\beta^{\frac{1}{\sigma}}R^{\frac{1}{\sigma}} + R}{\beta^{\frac{1}{\sigma}}\rho^{\frac{1}{\sigma}} + R}\right) > 1.$$

Note that both subsidy and pension tax increase with the level of imperfection in the education loan market with $s^*, p^* = 0$ when there is no imperfection ($\rho = R$).

7. Time consistency problem

In the paper we have assumed an implicit contract concerning intra-family intergenerational transfers between children and parents (expenses on education from parents to children and old age support from children to parents) and that the contract will be honored. However, there might exist a prisoner's dilemma kind of situation where agents renounce the contract in favor of a dominant strategy. The existence of a "family constitution", which is sustainable and renegotiation-proof under certain assumptions with agents optimizing individually subject to the rules of the constitution, has been the focus of many papers. Cigno (1993), for example, establishes conditions under which the contract is self-enforcing in the sense that it supports a subgame-perfect Nash equilibrium.[31]

Ehrlich and Lui (1991) argue that such a contract between parents and children is self-enforcing if a deviation entails similar violation from the subsequent generations and consumption in the middle and old age are sufficiently non-substitutable. In this section we show that, given these assumptions, the contract is self-enforcing as well as time consistent in our framework where parents are altruistic. We also observe that the self-enforcing nature of the contract is independent of the government policy regarding subsidies and pensions.

For ease of exposition, we assume a CRRA utility function as presented above. Define, for convenience, $M = n_t(1-s_t)(e_t(1-b_t)+\gamma)$, $B = H_{t+1}d_{t+1}n_t - b_te_tn_t(1-s_t)\rho$, and $A = n_t^{\phi}H_{t+1}^{\psi}$. Here, M, B and A denote the net cost incurred on children in the middle age, net material benefit reaped from children in the old age, and the altruism benefit/ companionship from children respectively. We assume that, as per the terms of the arrangement, if a parent does not honor her terms of the contract of providing old age support to her parents or providing human capital support to her children, it results in the violation of the contract from all the generations after her. This arrangement implies that the violation of the contract leads to a loss of material as well as altruistic benefits from children.

If an agent in her middle age *honors* her terms of the contract by giving old age support to her parents and incurring the education expenses on the children, and her children honor their terms by supporting her in her old age, then her utility is given by

$$\mathcal{U}^h = \frac{\big(a_t + H_t(1-d_t) - \tau_t - M\big)^{1-\sigma}}{1-\sigma} + \frac{\beta\big(B + P_{t+1}\big)^{1-\sigma}}{1-\sigma} + \frac{\delta(A)^{1-\sigma}}{1-\sigma}.$$

When the agent *reneges* on her contract, she loses material and emotional support from her children[32], leaving her with no incentive to invest in them. Her utility, in that case, is given by

$$\mathcal{U}^r = \frac{(a_t + H_t - \tau_t)^{1-\sigma}}{1-\sigma} + \frac{\beta P_{t+1}^{1-\sigma}}{1-\sigma}.$$

When σ is sufficiently large so that consumption is sufficiently non-substitutable intertemporally, we have $\mathcal{U}^h > \mathcal{U}^r$.[33]

[31] Rosati (1996) adds uncertainty to the basic framework. Cigno and Rosati (2000) introduce personal services without a perfect market substitute. Anderberg and Balestrino (2003) introduce education. Cigno (2006) introduces altruism while Cigno et al. (2017) bring in heterogeneity and marriage. These papers also show how the existence of family constitutions affects individuals' response to policy.

[32] Emotional support is not crucial to implement the contract. The contract is implementable with a sufficiently high σ even when δ is 0.

[33] Note that here parents do not have access to a savings technology that will allow them to smoothen consumption across time periods. Our results should go through even if we allow for savings as long as the interest rate on savings is not too high.

As argued by Ehrlich and Lui (1991), the fundamental reason behind the self-enforcement of the contract is the threat of losing old age benefits. The trigger strategy of violation by the next and the following generations ensures enforcement in equilibrium. Also, the old-age transfer ratio being set to maximize children's as well as parents' expected utilities, neither parties have the incentive to deviate from the terms of the contract. Moreover, the parental altruism towards children make deviation from existing contract even more costly. Since an infinite time horizon characterizes our economy, this results in the time consistency of the contract.

8. Human capital externality

In the analysis so far we have assumed that the human capital production function is free from any externalities. Education is sub-optimal due to imperfection in the education loan market, which provides the sole reason for government intervention. However, there is a huge body of literature which emphasizes the role of aggregate human capital externality in providing rationale for state intervention.[34] Hence, another reason for education subsidy can be the social benefit that the average level of human capital in the economy generates, the one the atomistic agents fail to internalize while taking the education decision.

In this section we assume that the human capital production function takes the form $H_{t+1} = h(\bar{H} + e_t)^{\alpha}\hat{H}^{\theta}_{t+1}$, where $\hat{H}_{t+1}$ is the average human capital of the middle aged agents in the economy in time period $t+1$. Here the externality has been defined in terms of average human capital as in Tamura (1991), Lucas (1988). We assume $\alpha + \theta < 1$ to rule out growth.[35] To focus on human capital externality, we assume away the credit market imperfection. To simplify the mechanics of the analysis, we assume further that the optimal old-age transfer proportion d is exogenous.

Similar to Section 3, education under *laissez-faire* (e_t^{CM}) is given by the implicit function

$$R(\gamma Z - \bar{H}) + dH^{CM}_{t+1}(\alpha - Z) - Re^{CM}_t(1 - Z) = 0, \tag{19}$$

whereas, as shown in Appendix F.1, the optimal education e_t^* (where parents internalize the externality while taking the education decision) is given by

$$R(\gamma Z - (1-\theta)\bar{H}) + dH^*_{t+1}(\alpha - Z) - Re^*_t(1 - \theta - Z) = 0. \tag{20}$$

Note that the investment in education under *laissez-faire* is less than the optimal education iff $\theta > 0$.

Suppose that the government intervenes in the education market by introducing a subsidy in the manner discussed in Section 4. Education under such intervention (e_t^E) is given by

$$R(1 - s_t)(\gamma Z - \bar{H}) + dH^E_{t+1}(\alpha - Z) - R(1 - s_t)e^E_t(1 - Z) = 0. \tag{21}$$

The investment in education is increasing in the level of subsidy s_t. The government can implement a subsidy such that the education in the economy replicates the optimal level, that is, $e^E(s_t) = e^*_t$. We call the level of subsidy that equates the optimal education and education under government intervention the "optimal subsidy" and denote it by s^*_t where

$$s^*_t = \frac{\theta(e^*_t + \bar{H})}{(1 - Z)e^*_t - (\gamma Z - \bar{H})}. \tag{22}$$

The level of optimal subsidy increases with the amount of externality (captured by the parameter θ).

Once the optimal subsidy is in place, the government is able to achieve the optimal education while fertility is decided by the following condition:

$$\beta u'(c^o_{t+1})\left(R(1 - s^*_t)(e^*{}_t + \gamma) - dH^*_{t+1}\right) = \delta u'(n^{\phi}_t H^{*\psi}_{t+1})n^{\phi-1}_t \phi H^{*\psi}_{t+1}. \tag{23}$$

Similar to the credit-market imperfection case, there are two channels via which fertility is impacted here – the *effective cost channel* and the *borrowing channel*. Via its impact on the effective cost, subsidy brings down the per child cost making the children less expensive. Via the same channel, it also decreases the loan repayment burden in the old age making the children more attractive on the margin. However, the subsidy leads to an upward adjustment in borrowing which makes children less attractive at the same time. In Appendix F.2, we show that, with the subsidy in place, the *effective cost channel* dominates the *borrowing channel*, leading to an increase in fertility beyond the optimal level. We also show that the government can use the negative pension-fertility relationship to bring the fertility down to its optimal level. This has been summarized in the following proposition.

[34] Human capital externality has been extensively analyzed since it has serious policy implications. Some of the important papers that deal with human capital externalities are Romer (1986), Lucas (1988), Azariadis and Drazen (1990), Tamura (1991), Barro et al. (1995), Benabou (1996), Heckman and Klenow (1998), Rudd (2000), Acemoglu and Angrist (2001), Moretti (2004), Ciccone and Peri (2006), among others.

[35] An intriguing avenue for future research would be to explore the role of pensions in a model that considers human capital investment as the engine for growth, as in Glomm and Ravikumar (1992). Further, in a model with income inequality, it would also be interesting to investigate the role of pensions as a balancing instrument. However, due to the scope limitations of the current paper, these aspects have not been addressed.

Proposition 3. *If the rationale for education subsidy is the presence of human capital externalities instead of credit market imperfection, Proposition 2 still holds. To be precise, education subsidy alone leads to overshooting of fertility, and, using an Education-Pension (EP) package, the government can replicate the complete-market levels of both education and fertility.*

When the market fails due to credit-market imperfections, the effective cost of raising children is sub-optimally high, and the education subsidy is engineered to bring the effective cost down to its complete-market level ($\rho(1-s^*) = R$). When the market fails due to the presence of human capital externality, the effective cost of raising children is already 'correct,' but problem lies in the internalized human capital returns from children which are lower than the actual social returns. Since, via subsidy, the government can alter the cost only and not the (internalized) returns, it responds with a subsidy which brings down the effective cost of children below the complete-market level ($R(1-s^*) < R$). In this case, the subsidy is engineered in such a way that the distortions from (sub-optimally) low effective cost and (sub-optimally) low returns cancel each other out, restoring education to the optimal level.

However, this tinkering with the effective cost of children ends up creating unintended distortion in the fertility choice of the agents. Externality creates a divergence between social and private returns from education directly, but the returns from fertility are impacted indirectly only via education. Hence, when subsidy restores education, returns from fertility stand corrected. However, the effective cost of children remains below the complete-market level due to government intervention. Even if we ignore the *borrowing channel*, this alone leads fertility to overshoot the optimal level.

In both the models, there is an indirect effect on old-age income working via borrowing. In the case of credit-market imperfections, government intervention in education fails to correct the high cost of borrowing, which causes borrowing to be sub-optimally low, making children less expensive on the margin. In the case of human capital externalities, the government, via subsidy, ends up creating additional income effects which affect consumption smoothing behavior of the agents. As old-age income increases due to lower repayment burden, agents respond by adjusting borrowing. They borrow more which makes children more expensive on the margin. However, as has been shown in both the models, the impact via borrowing is only secondary, with the *effective cost channel* dominating wherever it is operative.

Hence, we see that whenever education and fertility are joint decisions, state intervention in one shall have repercussions for the other. Irrespective of the motivation for government intervention, any attempt to alter trade-offs in education will sub-optimally alter trade-offs in fertility. We see that the intervention affects fertility either via its impact on the net cost of children or adjustment in the consumption smoothing behavior of the agents. Irrespective of the choice of the models, intervention in education necessarily leads to overshooting via these channels. The negative pension-fertility relation shown in Proposition 3 acts via only income effects and hence is independent of the nature of the model. The government can implement an Education-Pension package to bring fertility back to its optimal level, whenever it overshoots.

9. Conclusion

Imperfections in the loan market for education raise the effective cost of borrowing. This results in under-investment in human capital, providing rationale for state intervention in education. The government, with the help of an education subsidy, can achieve the optimal level of investment in education. The government is able to correct the underinvestment in education because, even though the agents finance the subsidy themselves, they do not internalize the higher tax burden due to investment in education. This atomistic behavior leads to an increase in the education level.

Investment in education (quality of children) is however very much interlinked with the choice of fertility (quantity of children). Any analysis of an optimal education subsidy that ignores the optimal fertility choice overlooks this important interdependence. To be precise, when education and fertility are joint decisions, state intervention only in education distorts the fertility choice. An education subsidy alters the trade-offs in the fertility decision, which causes fertility to overshoot its optimum.

In a dynamically efficient economy, when parents are altruistic towards children and rely on them for old age support, introducing a pension program for the old leads to a fall in fertility in the steady state. This negative relationship between the state-provided old age support and the fertility level can be exploited by the government to cancel out distortions generated by the education subsidy. We show the existence of an Education-Pension package that can replicate the optimal levels of both education and fertility in the economy. The optimal Education-Pension package expands with the degree of market imperfection.

Another potential reason for the state intervention in education is human capital externalities. We extend our analysis by introducing such externalities. Out results are invariant to the reason for market failure. Irrespective of how one models the motivation for education subsidy, it causes fertility to exceed its optimum. The negative relation between pensions and fertility is independent of the choice of the model, and the government can use this negative relationship to restore fertility to its optimum.

Becker et al. (2016) pointed out, in countries with social security systems in place, many middle-class and even poorer elderly parents have enough income that they do not rely on help from their children. They report that, while in the US and various European nations less than 30 percent of children help out their elderly parents, this number exceeds 60 percent in many poorer countries, including India and China and more than 80 percent in Singapore. Given this backdrop, our findings are most relevant for countries where a vibrant social security is not yet in place. Our results suggest that an education-pension (EP) package not only acts as a substitute for a missing education credit market, but it can also be used to promote optimal fertility choices.

Appendix A. Laissez-Faire Investment in Education

A1. Optimal investment rule

Substituting the constraints into the objective function, the Lagrange for problem (2) is

$$\mathcal{L} = u(a_t + h(\bar{H} + e_{t-1})^{\alpha}(1 - d_t) - (e_t(1 - b_t) + \gamma)n_t) + \beta u(h(\bar{H} + e_t)^{\alpha} d_{t+1} n_t - e_t b_t n_t \rho) + \delta u(n_t^{\phi}(h(\bar{H} + e_t)^{\alpha})^{\psi}).$$

The FOCs are

$$\frac{\partial \mathcal{L}}{\partial e_t} : \beta u'(c^o_{t+1})\left(d_{t+1} n_t \alpha \frac{H_{t+1}}{\bar{H} + e_t} - b_t n_t \rho\right) + \delta u'(n_t^{\phi} H_{t+1}^{\psi})\left(n_t^{\phi} \psi \alpha \frac{H_{t+1}^{\psi}}{\bar{H} + e_t}\right) - u'(c_t^m)(1 - b_t)n_t \le 0, \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} : \beta u'(c^o_{t+1})\left(d_{t+1} H_{t+1} - b_t e_t \rho\right) + \delta u'(n_t^{\phi} H_{t+1}^{\psi})\left(\phi n_t^{\phi-1} H_{t+1}^{\psi}\right) - u'(c_t^m)\left((1 - b_t)e_t + \gamma\right) \le 0, \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial b_t} : n_t e_t\left(u'(c_t^m) - \beta \rho u'(c^o_{t+1})\right) \le 0. \tag{A.3}$$

Assuming interior solution and substituting (A.3) in (A.1) and (A.2) the conditions become

$$\beta u'(c^o_{t+1})\left(\rho n_t - d_{t+1} n_t \alpha \frac{H_{t+1}}{\bar{H} + e_t}\right) = \delta u'(n_t^{\phi} H_{t+1}^{\psi})\left(n_t^{\phi} \psi \alpha \frac{H_{t+1}^{\psi}}{\bar{H} + e_t}\right), \tag{A.4}$$

$$\beta u'(c^o_{t+1})\left(\rho(e_t + \gamma) - d_{t+1} H_{t+1}\right) = \delta u'(n_t^{\phi} H_{t+1}^{\psi})\left(\phi n_t^{\phi-1} H_{t+1}^{\psi}\right). \tag{A.5}$$

Dividing the two, we get

$$\frac{\rho(\bar{H} + e_t) - d_{t+1}\alpha H_{t+1}}{\rho(e_t + \gamma) - d_{t+1} H_{t+1}} = \frac{\psi \alpha}{\phi} (\equiv Z).$$

Rearranging, we get

$$\rho(\gamma Z - \bar{H}) + d_{t+1} H_{t+1}(\alpha - Z) - \rho e_t (1 - Z) = 0.$$

We assume $\bar{H}$ to be small to allow for an interior solution in e_t.

A2. Optimal old-age transfer proportion

The parent chooses d_{t+1} so as to maximize $\mathcal{U}_{t+1}$ taking the optimal investment rule as given:

$$\max_{d_{t+1}} u(c^m_{t+1}) + \beta u(c^o_{t+2}) + \delta u(n_{t+1}^{\phi} H_{t+2}^{\psi})$$

subject to

$$c^m_{t+1} = a_{t+1} + H_{t+1}(1 - d_{t+1}) - e_{t+1}\big((1 - b_{t+1})n_{t+1} + \gamma\big),$$
$$c^o_{t+2} = H_{t+2} d_{t+2} n_{t+1} - e_{t+1} b_{t+1} n_{t+1} \rho,$$
$$H_{t+2} = h(\bar{H} + e_{t+1})^{\alpha},$$
$$\rho(\gamma Z - \bar{H}) + d_{t+1} H_{t+1}(\alpha - Z) - \rho e_t (1 - Z) = 0.$$

Substituting all the constraints and differentiating with respect to d_{t+1} yields the following FOC:

$$\frac{d\mathcal{U}_{t+1}}{dd_{t+1}} = \frac{\partial \mathcal{U}_{t+1}}{\partial d_{t+1}} + \frac{\partial \mathcal{U}_{t+1}}{\partial H_{t+1}} \frac{dH_{t+1}}{dd_{t+1}} + \left(\frac{\partial \mathcal{U}_{t+1}}{\partial n_{t+1}} \frac{dn_{t+1}}{dd_{t+1}} + \frac{\partial \mathcal{U}_{t+1}}{\partial b_{t+1}} \frac{db_{t+1}}{dd_{t+1}} + \frac{\partial \mathcal{U}_{t+1}}{\partial e_{t+1}} \frac{de_{t+1}}{dd_{t+1}}\right) = 0 \quad .$$

The envelope condition implies that the expression in the brackets is equal to zero. After substituting for the value of first two terms, the equation becomes

$$-u'(c^m_{t+1})H_{t+1} + u'(c^m_{t+1})(1 - d_{t+1})h\alpha(\bar{H} + e_t)^{\alpha-1} \frac{H_{t+1}}{\rho(\frac{1-Z}{\alpha-Z}) - d_{t+1}\alpha \frac{H_{t+1}}{\bar{H}+e_t}} = 0.$$

Simplifying further, we get

$$d_{t+1} = \alpha\left(1 - \frac{(\gamma - \bar{H})Z}{(e_t + \bar{H})(1 - Z)}\right), \tag{A.6}$$

$$\bar{H} + e_t = \left(\frac{\alpha h(\alpha - Z)}{\rho(1 - Z)}\right)^{\frac{1}{1-\alpha}}. \tag{A.7}$$

Appendix B. Investment in Education with Government Subsidy

B1. Optimal investment rule

After substituting the constraints into the objective function, the Lagrange for problem (10) becomes

$$\mathcal{L} = u(a_t + H_t(1 - d_t) - \big(e_t(1 - b_t) + \gamma\big)(1 - s_t)n_t - \tau_t) + \beta u(H_{t+1}d_{t+1}n_t - e_t(1 - s_t)b_t n_t \rho) + \delta u(n_t^{\phi} H_{t+1}^{\psi}).$$

The FOCs for this problem are

$$\frac{\partial \mathcal{L}}{\partial e_t} : \beta u'(c_{t+1}^o)n_t\left(d_{t+1}\alpha\frac{H_{t+1}}{\bar{H} + e_t} - b_t(1 - s_t)\rho\right) + \delta u'(n_t^{\phi}H_{t+1}^{\psi})\left(n_t^{\phi}\alpha\psi\frac{H_{t+1}^{\psi}}{\bar{H} + e_t}\right) - u'(c_t^m)(1 - s_t)(1 - b_t)n_t \le 0, \tag{B.1}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} : \beta u'(c_{t+1}^o)\left(d_{t+1}H_{t+1} - b_t e_t(1 - s_t)\rho\right) + \delta u'(n_t^{\phi}H_{t+1}^{\psi})\left(\phi n_t^{\phi-1}H_{t+1}^{\psi}\right) - u'(c_t^m)(1 - s_t)\big((1 - b_t)e_t + \gamma\big) \le 0, \tag{B.2}$$

$$\frac{\partial \mathcal{L}}{\partial b_t} : n_t(1 - s_t)e_t\left(u'(c_t^m) - \beta\rho u'(c_{t+1}^o)\right) \le 0. \tag{B.3}$$

Assuming interior solution and substituting (B.3) in (B.1) and (B.2) the conditions become

$$\beta u'(c_{t+1}^o)\left((1 - s_t)\rho n_t - d_{t+1}n_t\alpha\frac{H_{t+1}}{\bar{H} + e_t}\right) = \delta u'(n_t^{\phi}H_{t+1}^{\psi})\left(\alpha\psi n_t^{\phi}\frac{H_{t+1}^{\psi}}{\bar{H} + e_t}\right), \tag{B.4}$$

$$\beta u'(c_{t+1}^o)\left((1 - s_t)\rho(e_t + \gamma) - d_{t+1}H_{t+1}\right) = \delta u'(n_t^{\phi}H_{t+1}^{\psi})\left(\phi n_t^{\phi-1}H_{t+1}^{\psi}\right). \tag{B.5}$$

Dividing the two, we get

$$\frac{\rho(1 - s_t)(\bar{H} + e_t) - d_{t+1}\alpha H_{t+1}}{\rho(1 - s_t)(e_t + \gamma) - d_{t+1}H_{t+1}} = \frac{\psi\alpha}{\phi}(\equiv Z).$$

Rearranging, we get

$$\rho(1 - s_t)(\gamma Z - \bar{H}) + d_{t+1}H_{t+1}(\alpha - Z) - \rho(1 - s_t)e_t(1 - Z) = 0. \tag{B.6}$$

B2. Direct optimal old-age transfer proportion

The parent chooses d_{t+1} so as to maximize $\mathcal{U}_{t+1}$ taking the optimal investment rule as given:

$$\max_{d_{t+1}} u(c_{t+1}^m) + \beta u(c_{t+2}^o) + \delta u(n_{t+1}^{\phi}H_{t+2}^{\psi}),$$

subject to

$$c_{t+1}^m = a_{t+1} + H_{t+1}(1 - d_{t+1}) - \big(e_{t+1}(1 - b_{t+1}) + \gamma\big)(1 - s_{t+1})n_{t+1} - \tau_{t+1},$$
$$c_{t+2}^o = H_{t+2}d_{t+2}n_{t+1} - e_{t+1}b_{t+1}(1 - s_{t+1})n_{t+1}\rho,$$
$$H_{t+2} = h(\bar{H} + e_{t+1})^{\alpha},$$
$$\rho(1 - s_t)(\gamma Z - \bar{H}) + d_{t+1}H_{t+1}(\alpha - Z) - \rho(1 - s_t)e_t(1 - Z) = 0.$$

Substituting all the constraints, differentiating with respect to d_{t+1} and using the envelope condition yields the following FOC:

$$-u'(c^m_{t+1})H_{t+1} + u'(c^m_{t+1})(1-d_{t+1})h\alpha(\bar{H}+e_t)^{\alpha-1}\frac{H_{t+1}}{\rho(1-s_t)(\frac{1-Z}{\alpha-Z}) - d_{t+1}\alpha\frac{H_{t+1}}{\bar{H}+e_t}} = 0.$$

Simplifying this equation gives

$$d_{t+1} = \alpha\left(1 - \frac{(\gamma-\bar{H})Z}{(e_t+\bar{H})(1-Z)}\right),$$
$$\bar{H}+e_t = \left(\alpha\frac{h(\alpha-Z)}{\rho(1-s_t)(1-Z)}\right)^{\frac{1}{1-\alpha}}.$$

Appendix C. Proof of Proposition 1

Proof. We proceed in two steps. In the first step, we show that the fertility under *laissez-faire* is less than the complete-market level. In the second step, we show that the government intervention which corrects distortion in education causes fertility to overshoot the complete-market level.

Step 1: *Fertility under laissez-faire (n^{IM}) is less than fertility under complete market (n^{CM}).*

Proof. When education loan market is incomplete, and there is no intervention from the government, first order conditions for n_t and b_t given by equations (A.5) and (A.3), reduced form equilibrium expressions for d_t and e_t given by equations (A.6) and (A.7), budget constraints for middle and old-age consumption along with the human capital production function determine the *laissez-faire* levels of $n_t, b_t, d_t, e_t, c^m_t, c^o_t$ and H_{t+1}.

Thus, the steady state *laissez-faire* levels of fertlity (n) and borrowing (b) are determined by the following system of equations:

$$\mathcal{F}(n,b;\rho) \equiv \delta u'(n^{\phi}(H^{IM})^{\psi})\left(\phi n^{\phi-1}(H^{IM})^{\psi}\right) - \beta u'(c^o)\left(\rho(e^{IM}+\gamma) - d^{IM}H^{IM}\right) = 0, \tag{C.1}$$

$$\mathcal{G}(n,b;\rho) \equiv u'(c^m) - \beta\rho u'(c^o) = 0, \tag{C.2}$$

where

$$c^m = a + H^{IM}(1-d^{IM}) - n\left(e^{IM}(1-b)+\gamma\right), \tag{C.3}$$

$$c^o = nH^{IM}d^{IM} - \rho e^{IM}nb, \tag{C.4}$$

$$d^{IM} = \alpha\left(1 - \frac{(\gamma-\bar{H})Z}{(e^{IM}+\bar{H})(1-Z)}\right), \tag{C.5}$$

$$H^{IM} = h(\bar{H}+e^{IM})^{\alpha}, \tag{C.6}$$

$$(e^{IM}+\bar{H})^{1-\alpha} = \frac{\alpha h(\alpha-Z)}{\rho(1-Z)}. \tag{C.7}$$

From here on, we drop the superscript *IM* for convenience. Equation (C.7) defines e^{IM} as a function of ρ, where $\frac{de^{IM}}{d\rho} < 0$. We define $\Lambda \equiv \frac{de^{IM}}{d\rho}$. Further we define

$$C \equiv \rho(e+\gamma) - dH > 0,$$
$$C' \equiv \frac{\partial C}{\partial e} = \rho Z\left(\frac{\alpha-1}{\alpha-Z}\right)\left(\frac{e+\gamma}{e+\bar{H}}\right) < 0,$$

$$\Psi \equiv \frac{\partial H(1-d)}{\partial e} = \frac{H}{e+\bar{H}}\bigg(\alpha(1-d)+(\alpha-d)\bigg) > 0,$$

$$\Theta \equiv \delta\phi\alpha\psi n^{\phi-1}\frac{H^{\psi}}{e+\bar{H}}u'(n^{\phi}H^{\psi})\left(1+\frac{u''(n^{\phi}H^{\psi})n^{\phi}H^{\psi}}{u'(n^{\phi}H^{\psi})}\right) > 0,$$

$$\Omega \equiv \delta\phi n^{\phi-2}H^{\psi}u'(n^{\phi}H^{\psi})\left((\phi-1)+\phi\frac{u''(n^{\phi}H^{\psi})n^{\phi}H^{\psi}}{u'(n^{\phi}H^{\psi})}\right) < 0.$$

Here, we are assuming that the non-material marginal benefit from children goes up as education per child increases. C is the net pecuniary cost per child. C' is the marginal increase in the net pecuniary cost of children due to an additional unit of investment in child's education. Ψ is the marginal increase in *net* labour income due to an extra unit of own education. Θ and Ω capture the change in marginal altruism benefit from children due to an additional child and additional unit of education respectively. Substituting (C.3)-(C.7) in (C.1) and (C.2), and differentiating, we get

$$\frac{\partial\mathcal{F}}{\partial b} = \beta\rho enCu''(c^{o}) < 0, \qquad \frac{\partial\mathcal{F}}{\partial n} = \Omega - \beta u''(c^{o})C(dH-\rho be),$$

$$\frac{\partial\mathcal{G}}{\partial b} = ne\big(u''(c^{m})+\beta\rho^{2}u''(c^{o})\big) < 0, \qquad \frac{\partial\mathcal{G}}{\partial n} = -u''(c^{m})\big(e(1-b)+\gamma\big) - \beta\rho u''(c^{o})(dH-\rho be) > 0,$$

$$\frac{\partial\mathcal{F}}{\partial\rho} = \beta\bigg(u''(c^{o})benC - u'(c^{o})(e+\gamma)\bigg) + \Lambda\bigg[\Theta - \beta\bigg(Cu''(c^{o})\big((1-b)-nC'\big)+u'(c^{o})C'\bigg)\bigg] < 0,$$

$$\frac{\partial\mathcal{G}}{\partial\rho} = \beta\bigg(u''(c^{o})ben\rho - u'(c^{o})\bigg) + \Lambda\bigg[u''(c^{m})\bigg(\Psi - n(1-b)\bigg) - \beta n\rho u''(c^{o})\bigg((1-b)-\rho C'\bigg)\bigg] < 0.$$

Note that

$$\begin{vmatrix} \frac{\partial\mathcal{F}}{\partial b} & \frac{\partial\mathcal{F}}{\partial n} \\ \frac{\partial\mathcal{G}}{\partial b} & \frac{\partial\mathcal{G}}{\partial n} \end{vmatrix} = \frac{\partial\mathcal{F}}{\partial b}\frac{\partial\mathcal{G}}{\partial n} - \frac{\partial\mathcal{G}}{\partial b}\frac{\partial\mathcal{F}}{\partial n} = -\frac{\partial\mathcal{F}}{\partial b}Cu''(c^{m}) - \frac{\partial\mathcal{G}}{\partial b}\Omega < 0,$$

where the last inequality follows from the concavity of $u(.)$ and the signs of $C, \Omega, \frac{\partial\mathcal{F}}{\partial b}$ and $\frac{\partial\mathcal{G}}{\partial b}$. Since $\begin{pmatrix} \frac{\partial\mathcal{F}}{\partial b} & \frac{\partial\mathcal{F}}{\partial n} \\ \frac{\partial\mathcal{G}}{\partial b} & \frac{\partial\mathcal{G}}{\partial n} \end{pmatrix}$ is invertible, we invoke implicit function theorm to say that given (C.3)-(C.7), the equations (C.1) and (C.2) define n and b as implicit functions of ρ where

$$\frac{dn(\rho)}{d\rho} = -\frac{\frac{\partial\mathcal{F}}{\partial b}\frac{\partial\mathcal{G}}{\partial\rho} - \frac{\partial\mathcal{G}}{\partial b}\frac{\partial\mathcal{F}}{\partial\rho}}{\frac{\partial\mathcal{F}}{\partial b}\frac{\partial\mathcal{G}}{\partial n} - \frac{\partial\mathcal{G}}{\partial b}\frac{\partial\mathcal{F}}{\partial n}}. \tag{C.8}$$

Since we know that the denominator is strictly negative, we are interested in the sign of the numerator. $\frac{\partial\mathcal{F}}{\partial b}\left(-\frac{\partial\mathcal{G}}{\partial\rho}/\frac{\partial\mathcal{G}}{\partial b}\right) > 0$ captures the effect of imperfection on fertility via its impact on borrowing, while $\frac{\partial\mathcal{F}}{\partial\rho} < 0$ captures the effect via its impact on effective cost of educating children. Note that the two effects are in opposite directions. Increasing ρ increases fertility via *borrowing channel*, but reduces fertility via *effective cost channel*. In presence of imperfection, and no government intervention, the effective cost channel dominates the borrowing channel, so that fertility falls as imperfections in the loan market increase.

$$\frac{\partial\mathcal{F}}{\partial b}\frac{\partial\mathcal{G}}{\partial\rho} - \frac{\partial\mathcal{G}}{\partial b}\frac{\partial\mathcal{F}}{\partial\rho} = \beta ne\bigg[\beta\rho dHu'(c^{o})u''(c^{o}) + u''(c^{m})\bigg(u'(c^{o})(e+\gamma) - benCu''(c^{o})\bigg)\bigg] + \Lambda\bigg[\frac{\partial\mathcal{F}}{\partial b}u''(c^{m})\Psi - \frac{\partial\mathcal{G}}{\partial b}\bigg(\Theta - \beta u'(c^{o})C'\bigg) - \beta n^{2}CC'u''(c^{m})u''(c^{o})\bigg] < 0.$$

The inequality follows from signs of $\Psi, \Theta, C, C', \frac{\partial\mathcal{F}}{\partial b}$ and $\frac{\partial\mathcal{G}}{\partial b}$, and concavity of $u(.)$. Hence, we have $\frac{dn(\rho)}{d\rho} < 0$. Note that $n(\rho) = n^{IM}$ and $n(R) = n^{CM}$. Since $\rho > R$, we have $n^{IM} < n^{CM}$. □

Step 2: *Fertility under education-subsidy-only package (n^E) overshoots n^{CM}.*

Proof. With the optimal subsidy in place, old-age transfer proportion, education and human capital are at d^{CM}, e^{CM} and H^{CM} respectively, and are invariant to the level of imperfection in the loan market. Substituting the value of optimal subsidy, the FOC (B.5) can be rewritten as

$$\beta u'(c^o_{t+1})\Big(R(e^{CM}+\gamma)-d^{CM}H^{CM}\Big)=\delta u'(n_t^{\phi}(H^{CM})^{\psi})\Big(\phi n_t^{\phi-1}(H^{CM})^{\psi}\Big).$$

This equation together with FOC (B.3) and the budget constraints determine n_t, b_t, c^m_t and c^o_{t+1}. Steady state n and b are thus determined by the following system:

$$\mathcal{F}(n,b;\rho)\equiv\delta u'(n^{\phi}(H^{CM})^{\psi})\Big(\phi n^{\phi-1}(H^{CM})^{\psi}\Big)-\beta u'(c^o)\Big(R(e^{CM}+\gamma)-d^{CM}H^{CM}\Big)=0, \tag{C.9}$$

$$\mathcal{G}(n,b;\rho)\equiv u'(c^m)-\beta\rho u'(c^o)=0, \tag{C.10}$$

where $c^m=a+H^{CM}(1-d^{CM})-n\big(e^{CM}(1-b)+\gamma\big)$ and $c^o=nH^{CM}d^{CM}-Re^{CM}nb$. We drop the superscript CM for convenience.

Differentiating $\mathcal{F}$ and $\mathcal{G}$, we get

$$\frac{\partial\mathcal{F}}{\partial b}=\beta RenCu''(c^o)<0,\qquad \frac{\partial\mathcal{F}}{\partial n}=\Omega-\beta u''(c^o)C(dH-Rbe),$$

$$\frac{\partial\mathcal{G}}{\partial b}=ne\big(u''(c^m)+\beta\rho Ru''(c^o)\big)<0,\qquad \frac{\partial\mathcal{G}}{\partial n}=-u''(c^m)\big(e(1-b)+\gamma\big)-\beta\rho u''(c^o)(dH-Rbe)>0,$$

$$\frac{\partial\mathcal{F}}{\partial\rho}=0,\qquad \frac{\partial\mathcal{G}}{\partial\rho}=-\beta u'(c^o)<0,$$

where C is the *corrected* net pecuniary cost per child, that is, $C\equiv R(e+\gamma)-dH$. Ω is defined as before, capturing the change in marginal altruism benefit from children due to an additional child. Note that altruism is concave in children, that is, $\Omega<0$ and children are net material cost, that is, $C>0$.

Here too, we have $\frac{\partial\mathcal{F}}{\partial b}\frac{\partial\mathcal{G}}{\partial n}-\frac{\partial\mathcal{G}}{\partial b}\frac{\partial\mathcal{F}}{\partial n}=-\frac{\partial\mathcal{F}}{\partial b}Cu''(c^m)-\frac{\partial\mathcal{G}}{\partial b}\Omega<0$. Since $\begin{pmatrix}\frac{\partial\mathcal{F}}{\partial b} & \frac{\partial\mathcal{F}}{\partial n}\\ \frac{\partial\mathcal{G}}{\partial b} & \frac{\partial\mathcal{G}}{\partial n}\end{pmatrix}$ is invertible, we invoke implicit function theorm to say that the equations (C.9) and (C.10) define n and b as implicit functions of ρ where $\frac{dn(\rho)}{d\rho}$ is given by (C.8).

Note that, here, the education level (e^{CM}) as well as the net material cost of having children (C) stand corrected. This shuts down the *effective cost channel* (which dominates under *laissez-faire*) resulting in $\frac{\partial\mathcal{F}}{\partial\rho}=0$. But since the true cost of borrowing stays above R, we still have $\frac{\partial\mathcal{F}}{\partial b}\Big(-\frac{\partial\mathcal{G}}{\partial\rho}/\frac{\partial\mathcal{G}}{\partial b}\Big)>0$ and the *borrowing channel* remains operative. Thus, here we have $\frac{\partial n(\rho)}{\partial\rho}>0$.

In the system of equations (C.9) and (C.10), when $\rho=R$, optimal subsidy is 0 and n is at its complete-market level, that is, $s^*(R)=0$ and $n(R)=n^{CM}$. When education loan market is incomplete, government adjusts the optimal subsidy $s^*(\rho)$, so that the effective cost of education $\rho(1-s^*(\rho))$ stands corrected at R, but the government can not alter the true cost of borrowing ρ, which remains above R. Since $\rho>R$ and $n'(.)>0$, we have $n(\rho)>n(R)$, that is, $n^E>n^{CM}$. □

Hence, when market is incomplete, fertility is sub-optimally lower than the complete-market level, but the education-subsidy-only package results in higher than complete-market level of fertility.

Appendix D. Proof of Proposition 2

We use the following result to prove the proposition.

Lemma 1. *There exists a negative relationship between fertility and PAYG pensions in a dynamically efficient economy.*

Proof. In presence of optimal education subsidy and pension tax p, the steady state n and b are determined by the system of equations (C.9) and (C.10) where

$$c^m=a+H^{CM}(1-d^{CM})-n\big(e^{CM}(1-b)+\gamma\big)-p, \tag{D.1}$$

$$c^o = nH^{CM}d^{CM} - Re^{CM}nb + np. \tag{D.2}$$

Differentiating $\mathcal{F}$ and $\mathcal{G}$, we get

$$\frac{\partial\mathcal{F}}{\partial b} = \beta RenCu''(c^o) < 0, \qquad \frac{\partial\mathcal{F}}{\partial n} = \Omega - \beta u''(c^o)C(dH - Rbe + p),$$

$$\frac{\partial\mathcal{G}}{\partial b} = ne\big(u''(c^m) + \beta\rho Ru''(c^o)\big) < 0, \qquad \frac{\partial\mathcal{G}}{\partial n} = -u''(c^m)\big(e(1-b)+\gamma\big) - \beta\rho u''(c^o)(dH - Rbe + p) > 0,$$

$$\frac{\partial\mathcal{F}}{\partial p} = -\beta Cnu''(c^o) > 0, \qquad \frac{\partial\mathcal{G}}{\partial p} = -u''(c^m) - \beta\rho u''(c^o)n > 0.$$

Then

$$\frac{\partial\mathcal{F}}{\partial b}\frac{\partial\mathcal{G}}{\partial n} - \frac{\partial\mathcal{G}}{\partial b}\frac{\partial\mathcal{F}}{\partial n} = -\frac{\partial\mathcal{F}}{\partial b}(C-p)u''(c^m) - \frac{\partial\mathcal{G}}{\partial b}\Omega.$$

Note that with pension tax, children yield an additional benefit of higher total pension payment in future. Given a pension tax, higher number of children translates into a larger tax base to support agents in their old age. However, since this transfer is mediated by the government, agents do not internalize the effect of their fertility choice on the overall pool of pension support. Thus, the net pecuniary cost per child that the agents internalize is C while true per child cost is $C - p$.

We consider $p \in (0, C)$, such that the optimal fertility, that is, the fertility resulting from the agents' problem where they internalise the *tax base* externality, is bounded. $p \in (0, C)$ is a sufficient condition for the sign of the expression to be negative, implying that $\begin{pmatrix} \frac{\partial\mathcal{F}}{\partial b} & \frac{\partial\mathcal{F}}{\partial n} \\ \frac{\partial\mathcal{G}}{\partial b} & \frac{\partial\mathcal{G}}{\partial n} \end{pmatrix}$ is invertible.

This allows us to invoke the implicit function theorm and say that given (D.1) and (D.2), (C.9) and (C.10) define n and b as implicit functions of p where

$$\frac{dn}{dp} = -\frac{\frac{\partial\mathcal{F}}{\partial b}\frac{\partial\mathcal{G}}{\partial p} - \frac{\partial\mathcal{G}}{\partial b}\frac{\partial\mathcal{F}}{\partial p}}{\frac{\partial\mathcal{F}}{\partial b}\frac{\partial\mathcal{G}}{\partial n} - \frac{\partial\mathcal{G}}{\partial b}\frac{\partial\mathcal{F}}{\partial n}}. \tag{D.3}$$

Note that $\frac{\partial\mathcal{F}}{\partial b}\left(-\frac{\partial\mathcal{G}}{\partial p}/\frac{\partial\mathcal{G}}{\partial b}\right) < 0$ captures the effect of pensions on fertility via its impact on borrowing, while $\frac{\partial\mathcal{F}}{\partial p} > 0$ captures the direct income effect of pensions. The two effects are in opposite directions, but as shown below, when dynamic efficiency holds, the former dominates the latter.

$$\frac{\partial\mathcal{F}}{\partial b}\frac{\partial\mathcal{G}}{\partial p} - \frac{\partial\mathcal{G}}{\partial b}\frac{\partial\mathcal{F}}{\partial p} = -\beta enCu''(c^m)u''(c^o)\Big(R - n\Big) < 0.$$

Hence, when the economy is dynamically efficient, which implies $R > n$, we have $\frac{dn}{dp} < 0$, that is, *a negative relationship between fertility and social security*. □

Now we prove the main result. With an EP package in place, the FOC for n is

$$\delta u'(n^{\phi}(H^{CM})^{\psi})\Big(\phi n^{\phi-1}(H^{CM})^{\psi}\Big) = \beta u'(c^o)\Big(R(e^{CM} + \gamma) - d^{CM}H^{CM}\Big),$$

given

$$c^o = nH^{CM}d^{CM} - Re^{CM}nb^{EP} + np,$$

where b^{EP} is the borrowing level under the EP package. Under complete market, FOC for n is the same but with

$$c^o = nH^{CM}d^{CM} - Re^{CM}nb^{CM}.$$

It can be seen that with $p = Re(b^{EP} - b^{CM})$, FOC for fertility under EP package replicates the FOC under complete market.

As has already been shown in the proof of Lemma 1, system of equations (C.9) and (C.10) implicitly defines b^{EP} as a function of ρ and p where

$$\frac{db^{EP}}{dx} = -\frac{\frac{\partial\mathcal{G}}{\partial n}\frac{\partial\mathcal{F}}{\partial x} - \frac{\partial\mathcal{G}}{\partial x}\frac{\partial\mathcal{F}}{\partial n}}{\frac{\partial\mathcal{F}}{\partial b}\frac{\partial\mathcal{G}}{\partial n} - \frac{\partial\mathcal{G}}{\partial b}\frac{\partial\mathcal{F}}{\partial n}} \qquad \text{for } x \in \{\rho, p\}. \tag{D.4}$$

Thus, the EP package where $s = 1 - \frac{R}{\rho}$ and p is given by the equation

$$\mathcal{J}(p;\rho) \equiv p - Re(b^{EP}(p,\rho) - b^{CM}) = 0 \tag{D.5}$$

replicates complete-market level of education and fertility. Below we show that p defined by the equation (D.5) is indeed positive when the loan market is imperfect.

Differentiating (D.5), we have

$$\frac{\partial \mathcal{J}}{\partial p} = 1 - Re\left(\frac{\partial b^{EP}}{\partial p}\right) = \frac{u''(c^m)(R-n)e\dfrac{\partial \mathcal{F}}{\partial n}}{\dfrac{\partial \mathcal{F}}{\partial b}\dfrac{\partial \mathcal{G}}{\partial p} - \dfrac{\partial \mathcal{G}}{\partial b}\dfrac{\partial \mathcal{F}}{\partial p}}.$$

Note from equation (D.4), if $\frac{\partial \mathcal{F}}{\partial n} = 0$ then $\frac{db^{EP}}{d\rho} = 0$ (as $\frac{\partial \mathcal{F}}{\partial \rho} = 0$). As can be seen by comparing the FOC for n^{EP} and n^{CM}, this in turn implies that fertility is invariant to the level of imperfection, once optimal subsidy is in place and $p = 0$, which is a contradiction to Proposition 1. Hence, we must have $\frac{\partial \mathcal{F}}{\partial n} \neq 0$. Since we are assuming dynamic efficiency ($R > n$) and have already shown the denominator to be non-zero in proof of part (a), we have $\frac{\partial \mathcal{J}}{\partial P}$ to be non-zero. This allows us to invoke implicit function theorm and say that the equation (D.5) defines p as a function of ρ, where

$$\frac{\partial p(\rho)}{\partial \rho} = -\frac{\dfrac{\partial \mathcal{J}}{\partial \rho}}{\dfrac{\partial \mathcal{J}}{\partial p}} = \frac{Re\dfrac{\partial b^{EP}}{\partial \rho}}{1 - Re\dfrac{\partial b^{EP}}{\partial p}} = \frac{R\dfrac{\partial \mathcal{G}}{\partial \rho}}{(R-n)u''(c^m)} > 0.$$

Here the inequality follows from dynamic efficiency, concavity of $u(.)$ and the sign of $\frac{\partial \mathcal{G}}{\partial \rho}$. Note that $b^{EP}(0, R) = b^{CM}$. Hence, ($p = 0, \rho = R$) is a solution to the Eq. D.5. From the implicit function theorm, we know that p is a function of ρ, implying that for a particular value of ρ, we will have a unique value of p. Hence, we have $p(R) = 0$. Since under imperfect market, $\rho > R$, and we have shown that $p'(.) > 0$ and $p(R) = 0$, we have $p(\rho) > 0$.

Appendix E. Optimal EP Package under CRRA Utility

With CRRA utility function, the steady state n and b are determined by the equations

$$\beta(R(e+\gamma) - dH) - \delta\phi n^{\phi-\phi\sigma-1}H^{\psi-\psi\sigma}(ndH - benR)^{\sigma} = 0, \tag{E.1}$$

$$\left(\frac{ndH - benR + np}{a + H(1-d) - n(e(1-b)+\gamma) - p}\right)^{\sigma} = \beta\rho. \tag{E.2}$$

From (E.2), we get b as a function of n given by the following equation:

$$b = \frac{Hdn + np - (\beta\rho)^{\frac{1}{\sigma}}[a + H(1-\alpha) - n(e+\gamma) - p]}{en(\beta^{\frac{1}{\sigma}}\rho^{\frac{1}{\sigma}} + R)}.$$

Substituting this in (E.1) yields the following:

$$\mathcal{Q}(n;\rho,p) \equiv \beta\big(R(e+\gamma) - dH\big) - \delta\phi n^{\phi-\phi\sigma-1}H^{\psi-\psi\sigma}\left[\left(\frac{(\beta\rho)^{\frac{1}{\sigma}}}{(\beta\rho)^{\frac{1}{\sigma}} + R}\right)\big(\Gamma - p\big(R-n\big)\big)\right]^{\sigma} = 0,$$

where $\Gamma = ndH + R[a + H(1-\alpha) - n(e+\gamma)]$.

Note that the complete-market level of fertility can be obtained by setting $\mathcal{Q}(n; R, 0)$ to zero and thus, is given by the following condition:

$$\beta\big(R(e+\gamma) - dH\big) - \delta\phi n^{\phi-\phi\sigma-1}H^{\psi-\psi\sigma}\left[\left(\frac{(\beta R)^{\frac{1}{\sigma}}}{(\beta R)^{\frac{1}{\sigma}} + R}\right)\Gamma\right]^{\sigma} = 0.$$

We define n^{CM} as the level of fertility that solves the above equation. With an EP package where subsidy is optimal and pension tax is given by

$$p^* \equiv \frac{(K-1)\Gamma}{K(R - n^{CM})} \tag{E.3}$$

where

$$K \equiv \left(\frac{\rho}{R}\right)^{\frac{1}{\sigma}-1}\left(\frac{\beta^{\frac{1}{\sigma}}R^{\frac{1}{\sigma}} + R}{\beta^{\frac{1}{\sigma}}\rho^{\frac{1}{\sigma}} + R}\right) > 1, \tag{E.4}$$

the final FOC for n given by $\mathcal{Q}(n;\rho,p^*) = 0$ replicates equation $\mathcal{Q}(n;R,0) = 0$. Thus, an EP package with s^* subsidy and p^* pension achieves complete-market levels of education and fertility.

Appendix F. The Case with Human Capital Externality

F1. Optimal subsidy

The level of education when government provides only education subsidy s_t (as derived in Appendix B.1) is given by the implicit function

$$R(1-s_t)(\gamma Z-\bar{H})+dH^E_{t+1}(\alpha-Z)-R(1-s_t)e^E_t(1-Z)=0. \tag{F.1}$$

Here the agent is atomistic and does not internalize the externality generated by the average level of human capital in the economy. We define the optimal education (and fertility) level to be the one where this externality is internalized by the agent, and there is no government intervention. Agent's FOCs in that case would be

$$\beta u'(c^o_{t+1})\left(Rn_t-dn_t\frac{\alpha}{1-\theta}\frac{H_{t+1}}{\bar{H}+e_t}\right)=\delta u'(n_t^{\phi}H^{\psi}_{t+1})\left(\frac{\alpha}{1-\theta}\psi n_t^{\phi}\frac{H^{\psi}_{t+1}}{\bar{H}+e_t}\right), \tag{F.2}$$

$$\beta u'(c^o_{t+1})\left(R(e_t+\gamma)-dH_{t+1}\right)=\delta u'(n_t^{\phi}H^{\psi}_{t+1})\left(\phi n_t^{\phi-1}H^{\psi}_{t+1}\right), \tag{F.3}$$

$$u'(c^m_t)=\beta Ru'(c^o_{t+1}). \tag{F.4}$$

Dividing the equations (F.2) and (F.3), we get

$$\frac{R(1-\theta)(\bar{H}+e_t)-d_{t+1}\alpha H_{t+1}}{R(e_t+\gamma)-d_{t+1}H_{t+1}}=\frac{\psi\alpha}{\phi}\qquad(\equiv Z).$$

Rearranging the terms gives

$$R(\gamma Z-(1-\theta)\bar{H})+dH^*_{t+1}(\alpha-Z)-Re^*_t(1-\theta-Z)=0. \tag{F.5}$$

With an education subsidy given by

$$s_t=s^*_t\equiv\frac{\theta(e^*_t+\bar{H})}{(1-Z)e^*_t-(\gamma Z-\bar{H})}, \tag{F.6}$$

the condition for the atomistic agent (given by equation (F.1)) replicates the corresponding condition for optimal education (equation (F.5)), such that $e^E(s^*_t)=e^*_t$.

F2. Fertility under education-only package

With the optimal subsidy in place, education and human capital are at e^* and H^* respectively. Steady state n and b are going to be determined by the following equations:

$$\mathcal{F}(n,b;s^*)\equiv\delta u'(n^{\phi}(H^*)^{\psi})\left(\phi n^{\phi-1}(H^*)^{\psi}\right)-\beta u'(c^o)\left(R(1-s^*)(e^*+\gamma)-dH^*\right)=0, \tag{F.7}$$

$$\mathcal{G}(n,b;s^*)\equiv u'(c^m)-\beta Ru'(c^o)=0,$$

where $c^m=a+H^*(1-d)-n\big(e^*(1-b)+\gamma\big)$ and $c^o=nH^*d-R(1-s^*)e^*nb$. Differentiating, we have

$$\frac{\partial\mathcal{F}}{\partial b}=\beta RenCu''(c^o)<0,$$
$$\frac{\partial\mathcal{G}}{\partial b}=ne(1-s^*)\big(u''(c^m)+\beta R^2u''(c^o)\big)<0,$$
$$\frac{\partial\mathcal{F}}{\partial n}=\Omega-\beta u''(c^o)C(dH-R(1-s^*)be),$$
$$\frac{\partial\mathcal{G}}{\partial n}=-u''(c^m)(e(1-b)+\gamma\big)-\beta Ru''(c^o)(dH-R(1-s^*)be)>0,$$
$$\frac{\partial\mathcal{F}}{\partial s^*}=\beta R(e+\gamma)u'(c^o)-\beta(R(1-s^*)(e+\gamma)-dH)u''(c^o)Renb>0,$$
$$\frac{\partial\mathcal{G}}{\partial s^*}=-\beta Ru''(c^o)Renb>0,$$

where $C\equiv R(1-s^*)(e+\gamma)-dH$ and has the same interpretation as before.

Again, it can be shown that $\frac{\partial \mathcal{F}}{\partial b}\frac{\partial \mathcal{G}}{\partial n} - \frac{\partial \mathcal{G}}{\partial b}\frac{\partial \mathcal{F}}{\partial n} < 0$ allowing us to invoke the implicit function theorm, and say that n can be expressed as a function of s^*, where

$$\frac{dn}{ds^*} = -\frac{\frac{\partial \mathcal{F}}{\partial b}\frac{\partial \mathcal{G}}{\partial s^*} - \frac{\partial \mathcal{G}}{\partial b}\frac{\partial \mathcal{F}}{\partial s^*}}{\frac{\partial \mathcal{F}}{\partial b}\frac{\partial \mathcal{G}}{\partial n} - \frac{\partial \mathcal{G}}{\partial b}\frac{\partial \mathcal{F}}{\partial n}}. \tag{F.8}$$

Here, as before, fertility is affected via two channels – the *effective cost channel* and the *borrowing channel*. $\frac{\partial \mathcal{F}}{\partial s^*} > 0$ captures the direct effect via change in the children's effective cost, while $\frac{\partial \mathcal{F}}{\partial b}(-\frac{\partial \mathcal{G}}{\partial s^*}/\frac{\partial \mathcal{G}}{\partial b}) < 0$ captures the indirect effect via adjusments in borrowing. Subsidy brings the effective cost of children down, but it leads to income effects which prompts a response from agents in form of increased borrowing. Note that, as before, the two channels act in opposite directions in affecting fertility.

We show that, as was the case in Appendix D, the *effective cost channel* dominates the *borrowing channel*, causing fertility under the subsidy to overshoot the optimal level.

$$\frac{\partial \mathcal{F}}{\partial b}\frac{\partial \mathcal{G}}{\partial s^*} - \frac{\partial \mathcal{G}}{\partial b}\frac{\partial \mathcal{F}}{\partial s^*} = -en(1-s^*)\left(u''(c^m)\frac{\partial \mathcal{F}}{\partial s^*} + \beta R^3 u''(c^o)u'(c^o)(e+\gamma)\right) > 0,$$

where the inequality follows from sign of $\frac{\partial \mathcal{F}}{\partial s^*}$ and concavity of $u(.)$.

Thus, we have $\frac{dn}{ds^*} > 0$. Also, note that when $s^* = 0$, the system of equations defines the fertility at its optimal level (which would have prevailed if agents had internalized the education externality, and there were no government intervention). Hence, with human capital externality, optimal subsidy results in the fertility overshooting its optimal level.

Proof for the negative fertility-pension relation, in this case, is the same as that in Appendix D.

References

Acemoglu, D., Angrist, J., 2001. How Large are human-capital externalities? evidence from compulsory-schooling laws. In: NBER Macroeconomics Annual 2000, Volume 15. MIT Press, pp. 9–74. http://www.nber.org/chapters/c11054

Amuedo-Dorantes, C., Pozo, S., 2006. Remittances as insurance: evidence from mexican immigrants. J. Popul. Econ. 19 (2), 227–254. doi:10.1007/s00148-006-0079-6.

Anderberg, D., Balestrino, A., 2003. Self-enforcing intergenerational transfers and the provision of education. Economica 70 (277), 55–71. doi:10.1111/1468-0335.d01-47.

Andersen, T.M., Bhattacharya, J., 2017. The intergenerational welfare state and the rise and fall of pay-as-you-go pensions. Econ. J. 127 (602), 896–923. doi:10.1111/ecoj.12330.

Azariadis, C., Drazen, A., 1990. Threshold externalities in economic development. Q. J. Econ. 105 (2), 501. doi:10.2307/2937797.

Barro, R.J., Becker, G., 1988. A reformulation of the economic theory of fertility. Q. J. Econ. 103 (1), 1. doi:10.2307/1882640.

Barro, R.J., Becker, G., 1989. Fertility choice in a model of economic growth. Econometrica 57 (2), 481. doi:10.2307/1912563.

Barro, R.J., Mankiw, N.G., Sala-i Martin, X., 1995. Capital mobility in neoclassical models of growth. Am. Econ. Rev. 85 (1), 103–115. doi:10.3386/w4206.

Becker, G., 1960. An economic analysis of fertility. Demograph Econ. Change Dev. Countries 209–240. https://econpapers.repec.org/bookchap/nbrnberch/2387.htm

Becker, G., 1962. Investment in human capital: a theoretical analysis. J. Polit. Econ. 70 (5, Part 2), 9–49.

Becker, G., Lewis, H. G., 1973. On the Interaction between the Quantity and Quality of Children. 10.2307/1840425.

Becker, G., Murphy, K., Spenkuch, J., 2016. The manipulation of childrens preferences, old-Age support, and investment in childrens human capital. J. Labor. Econ. 34 (S2), S3–S30. doi:10.1086/683778.

Becker, G., Murphy, K.M., 1988. The family and the state. J. Law Econ. 31 (1), 1–18. http://www.jstor.org/stable/725451

Becker, G., Murphy, K.M., Tamura, R., 1990. Human capital, fertility, and economic growth. J. Polit. Econ. 98 (5, Part 2), S12–S37. doi:10.1086/261723.

Benabou, R., 1996. Equity and efficiency in human capital investment: the local connection. Rev. Econ. Stud. 63 (2), 237. doi:10.2307/2297851.

Bishnu, M., 2013. Linking consumption externalities with optimal accumulation of human and physical capital and intergenerational transfers. J. Econ. Theory 148 (2), 720–742. doi:10.1016/J.JET.2012.08.006.

Bishnu, M., Garg, S., Garg, T., Ray, T., 2021. Optimal intergenerational transfers: public education and pensions. J. Public Econ. 198, 104411. doi:10.1016/J.JPUBECO.2021.104411.

Bishnu, M., Wang, M., 2017. The political intergenerational welfare state J. Econ. Dyn. Control 77, 93–110. doi:10.1016/j.jedc.2017.02.002.

Boldrin, M., De Nardi, M., Jones, L., 2015. Fertility and social security. J. Demogr. Econ. 81 (03), 261–299. doi:10.1017/dem.2014.14.

Boldrin, M., Jones, L., 2002. Mortality, fertility, and saving in a malthusian economy. Rev. Econ. Dyn. 5 (4), 775–814. doi:10.1006/REDY.2002.0186.

Boldrin, M., Montes, A., 2005. The intergenerational state education and pensions. Rev. Econ. Stud. 72 (3), 651–664. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.589.3121&rep=rep1&type=pdf

Butz, W., Stan, P., 1982. Household composition and interhousehold exchange in Malaysia.https://www.popline.org/node/390163.

Chapman, B., 2006. Income contingent loans for higher education: international reforms. In: Handbook of the Economics of Education, Vol. 2. Elsevier, pp. 1435–1503. doi:10.1016/S1574-0692(06)02025-3.

Ciccone, A., Peri, G., 2006. Identifying human-Capital externalities: theory with applications. Rev. Econ. Stud. 73, 381–412. http://giovanniperi.ucdavis.edu/uploads/5/6/8/2/56826033/ciccone_peri_identifying_human_capital_2006.pdf

Cigno, A., 1986. Fertility and the tax-Benefit system: areconsideration of the theory of family taxation. Econ. J. 96 (384), 1035. doi:10.2307/2233172.

Cigno, A., 1993. Intergenerational transfers without altruism: family, market and state. Eur. J. Polit. Econ. 9 (4), 505–518. doi:10.1016/0176-2680(93)90036-T.

Cigno, A., 2006. A constitutional theory of the family. J. Popul. Econ. 19 (2), 259–283. doi:10.1007/s00148-006-0062-2.

Cigno, A., Casolaro, L., Rosati, F.C., 2003. The impact of social security on saving and fertility in germany. FinanzArchiv: Public Finance Anal. 59 (2), 189–211. doi:10.2307/40912996.

Cigno, A., Komura, M., Luporini, A., 2017. Self-enforcing family rules, marriage and the (non)neutrality of public intervention. J. Popul. Econ. 30 (3), 805–834. doi:10.1007/s00148-017-0639-y.

Cigno, A., Rosati, F.C., 1990. The effects of financial markets and social security on saving and fertility behaviour in italy. J. Popul. Econ. 5 (4), 319–341. doi:10.1007/bf00163064.
Cigno, A., Rosati, F.C., 1996. Jointly determined saving and fertility behaviour: theory, and estimates for germany, italy, UK and USA. Eur. Econ. Rev. 40 (8), 1561–1589. doi:10.1016/0014-2921(95)00046-1.
Cigno, A., Rosati, F.C., 2000. Mutual interest, self-enforcing constitutions and apparent generosity. IEA Conf. Vol. Ser. 130, 226–247.
Conde-Ruiz, J.I., Giménez, E.L., Pérez-Nievas, M., 2010. Millian efficiency with endogenous fertility 77 (1), 154–187. doi:10.1111/j.1467-937X.2009.00568.x.
Conesa, J.C., Krueger, D., 1999. Social security reform with heterogeneous agents 2 (4), 757–795. doi:10.1006/redy.1998.0039.
Córdoba, J.C., Ripoll, M., 2016. Intergenerational transfers and the fertilityincome relationship. Econ. J. 126 (593), 949–977. doi:10.1111/ECOJ.12197.
Cox, D., Jimenez, E., 1990. Achieving social objectives through private transfers: a review. World Bank Res. Obs. 5 (2), 205–218. doi:10.1093/wbro/5.2.205.
Cox, D., Jimenez, E., 1992. Social security and private transfers in developing countries: the case of peru. World Bank Econ. Rev. 6 (1), 155–169. doi:10.1093/wber/6.1.155.
De la Croix, D., Doepke, M., 2003. Inequality and growth: why differential fertility matters. Am. Econ. Rev. 93 (4), 1091–1113. doi:10.1257/000282803769206214.
De la Croix, D., Doepke, M., 2004. Public versus private education when differential fertility matters. J. Dev. Econ. 73 (2), 607–629. doi:10.1016/j.jdeveco.2003.05.005.
Daruich, D., Kozlowski, J., 2020. Explaining intergenerational mobility: the role of fertility and family transfers. Rev. Econ. Dyn. 36, 220–245. doi:10.1016/J.RED.2019.10.002.
De Fraja, G., 2002. The design of optimal education policies. Rev. Econ. Stud. 69 (2), 437–466.
Dee, T.S., 2004. Are there civic returns to education? J. Public Econ. 88 (9–10), 1697–1720. https://ideas.repec.org/a/eee/pubeco/v88y2004i9-10p1697-1720.html
Doepke, M., 2004. Accounting for fertility decline during the transition to growth. J. Econ. Growth 9 (3), 347–383. doi:10.1023/B:JOEG.0000038935.84627.e4.
Durlauf, S.N., Fafchamps, M., 2005. Social capital. Handbook Econ. Growth 1, Part B, 1639–1699. https://ideas.repec.org/h/eee/grochp/1-26.html
Ehrlich, I., Lui, F.T., 1991. Intergenerational trade, longevity, and economic growth. J. Polit. Econ. 99 (5), 1029–1059. doi:10.1086/261788.
Ehrlich, I., Lui, F.T., 1998. Social security, the family, and economics growth. Econ. Inq. 36 (3), 390–409. doi:10.1111/j.1465-7295.1998.tb01722.x.
Friedman, M., 1955. The role of government in education. Econ. Public Interest. http://homepage.fudan.edu.cn/jfeng/files/2011/08/role-of-government-in-education_Friedman.pdf
Friedman, M., 1962. Capitalism and Freedom. University of Chicago Press. http://www.press.uchicago.edu/ucp/books/book/chicago/C/bo18146821.html
Friedman, M., Kuznets, S., 1945. Income from Independent Professional Practice. New York
Galor, O., Zeira, J., 1993. Income distribution and macroeconomics. Rev. Econ. Stud. 60 (1), 35–52. http://links.jstor.org/sici?sici=0034-6527%28199301%2960%3A1%3C35%3AIDAM%3E2.0.CO%3B2-U
Glomm, G., Ravikumar, B., 1992. Public versus private investment in human capital: endogenous growth and income inequality10.1086/261841.
Golosov, M., Jones, L., Tertilt, M., 2007. Efficiency with endogenous population growth. Econometrica 75 (4), 1039–1071. doi:10.1111/j.1468-0262.2007.00781.x.
Gonzalez-Eiras, M., Niepelt, D., 2012. Ageing, government budgets, retirement, and growth. Eur. Econ. Rev. 56 (1), 97–115.
Hammond, P., 1975. Charity: altruism or cooperatice egoism? Phelps, E. S. (Ed.) Altruism, morality and economic theory, New York (Russell Sage Foundation) 115–132. https://opus4.kobv.de/opus4-Fromm/frontdoor/index/index/docId/11426
Heckman, J., Klenow, P., 1998. Human capital polict. Policies to Promote Capital Formation.
Hohm, C.F., 1975. Social security and fertility: an international perspective. Demography 12 (4), 629–644. https://about.jstor.org/terms
Jones, L. E., Tertilt, M., 2008. Chapter 5 an economic history of fertility in the United States: 18261960, 165–230. doi:10.1016/S1574-0129(08)00005-7.
Knowles, J.C., Anker, R., 1981. An analysis of income transfers in a developing country: the case of kenya. J. Dev. Econ. 8 (2), 205–226. doi:10.1016/0304-3878(81)90029-8.
Lancia, F., Russo, A., 2016. Public education and pensions in democracy: apolitical economy theory. J. Eur. Econ. Assoc. 14 (5), 1038–1073.
Leibenstein, H., 1957. Economic backwardness and economic growth. http://agris.fao.org/agris-search/search.do?recordID=US201300602487.
Leibenstein, H., 1975. The economic theory of fertility decline. Q. J. Econ. 89 (1), 1. doi:10.2307/1881706.
Looney, A., Lee, V., 2018. Parents are borrowing more and more to send their kids to collegeand many are struggling to repay. Technical Report. Brookings. https://www.brookings.edu/research/parents-are-borrowing-more-and-more-to-send-their-kids-to-college-and-many-are-struggling-to-repay/
Lucas, R.E., 1988. On the mechanics of economic development. J. Monet. Econ. 22, 3–42. https://www.parisschoolofeconomics.eu/docs/darcillon-thibault/lucasmechanicseconomicgrowth.pdf
Manuelli, R.E., Seshadri, A., 2009. Explaining international fertility differences. Q. J. Econ. 124 (2), 771–807. doi:10.1162/QJEC.2009.124.2.771.
Michel, P., Wigniolle, B., 2007. On efficient child making 31 (2), 307–326.
Moretti, E., 2004. Workers' Education, spillovers, and productivity: evidence from plant-Level production functions. Am. Econ. Rev. 94 (3), 656–690. doi:10.1257/0002828041464623.
Nerlove, M., 1975. Some problems in the use of income-contingent loans for the finance of higher education. J. Polit. Econ. 83 (1), 157–183. doi:10.1086/260311.
Nugent, J.B., 1985. The old-Age security motive for fertility. Popul. Dev. Rev. 11 (1), 75. doi:10.2307/1973379.
OECD, 2017. Education at a Glance 2017. Technical Report doi:10.1787/eag-2017-en.
OECD, 2017. Pensions at a Glance 2017: OECD and G20 Indicators. Technical Report doi:10.1787/pension_glance-2017-en.
Palacios, R., Sluchynsky, O., 2006. Social pensions part I: their role in the overall pension system. http://documents.worldbank.org/curated/en/116531468159898116/Social-pensions-Part-I-their-role-in-the-overall-pension-system.
Pallares-Miralles, M., Romero, C., Whitehouse, E., 2012. International Patterns of Pension Provision II : a Worldwide Overview of Facts and Figures. http://documents.worldbank.org/curated/en/143611468168560687/International-patterns-of-pension-provision-II-a-worldwide-overview-of-facts-and-figures.
Pogue, T.F., Sgontz, L.G., 1977. Social security and investment in human capital. Natl. Tax J. 30, 157–169. doi:10.2307/41862126.
Poterba, J., 1996. Demographic structure and the political economy of public education. 10.3386/w5677.
Poterba, J., 1998. Demographic change, intergenerational linkages, and public education. 10.2307/116940.
Putnam, R.D., Leonardi, R., Nanetti, R., 1993. Making democracy work : Civic traditions in modern italy. Princeton University Press. https://press.princeton.edu/titles/5105.html
Rangel, A., 2003. Forward and backward intergenerational goods: why is social security good for the environment? Am. Econ. Rev. 93 (3), 813–834. http://www.jstor.org/stable/3132118
Ravallion, M., Dearden, L., 1988. Social security in a "moral economy": an empirical analysis for java. Rev. Econ. Stat. 70 (1), 36. doi:10.2307/1928148.
Rempel, H., Lobdell, R.A., 1978. The role of urban to rural remittances in rural development. J. Dev. Stud. 14 (3), 324–341. doi:10.1080/00220387808421678.
Romer, P.M., 1986. Increasing returns and long-Run growth. J. Polit. Econ. 94 (5), 1002–1037. http://links.jstor.org/sici?sici=0022-3808%28198610%2994%3A5%3C1002%3AIRALG%3E2.0.CO%3B2-C
Rosati, F.C., 1996. Social security in a non-altruistic model with uncertainty and endogenous fertility. J. Public Econ. 60 (2), 283–294. doi:10.1016/0047-2727(95)01513-2.
Rudd, J., 2000. Empirical evidence on human capital spillovers. Finance Econ. Discuss. Ser. 2000-46 mimeo. https://www.federalreserve.gov/pubs/feds/2000/200046/200046pap.pdf
Schoonbroodt, A., Tertilt, M., 2014. Property rights and efficiency in OLG models with endogenous fertility 150, 551–582. doi:10.1016/j.jet.2013.09.016.
Stiglitz, J.E., Weiss, A., 1981. Credit rationing in markets with imperfect information. Am. Econ. Rev. 71 (3), 393–410. http://www.jstor.org/stable/1802787
Tamura, R., 1991. Income convergence in an endogeneous growth model. J. Polit. Econ. 99 (3), 522–540. doi:10.1086/261765.
Zilcha, I., 1991. Characterizing efficiency in stochastic overlapping generations models. J. Econ. Theory 55 (1), 1–16. doi:10.1016/0022-0531(91)90056-A.