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Optimal intergenerational transfers: Public education and pensions ☆

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ABSTRACT

In presence of imperfections in the education loan market, the standard policy response of intervening solely on the education front, funded through taxes and transfers, necessarily hurts the initial working population. The literature suggests compensating them via Pay-As-You-Go (PAYG) pensions as a possible solution. We carry out the optimal policy exercise of a utilitarian government in a dynamically efficient economy with pension and education support obeying the Pareto criterion. We find that expansion of one instrument along with the other emerges as the optimal response, however, once the complete market level of education is achieved, the optimal policy suggests phasing pensions out. Eventually, government leads the economy to an equilibrium with zero pension and the Golden Rule level of education. This is achieved by exploiting only market opportunities without relying on other factors including human capital externalities, general equilibrium effects, or socio-political factors. We complement our theoretical results with a numerical exercise and compute the optimal policy path under different initial conditions and parameter values.

1. Introduction

The education loan market in most countries is far from perfect. Sometimes the market does not exist, and even when it does it is often extremely thin. Typically in this situation, the government steps in with large education subsidy programs.[1] While these subsidies are enjoyed in the early periods of life, in many countries, individuals receive support in their old-age as well, in the form of pension. Education subsidy and Pay-As-You-Go (PAYG) pension are the two most significant support programs by governments around the world.[2] Generally, the working population is the financier of these transfers – taxes on their earnings are channeled to two different generations. An *education-pension policy package*, with a forward intergenerational arm in the form of education subsidies and backward intergenerational arm of pension support, can be viewed as a two-way policy of state-mediated intergenerational transfer.

The idea of linking backward and forward intergenerational goods is not new. Becker and Murphy (1988) link the parental investment in education and pension by considering them as trade among generations: children receive education from their parents and in exchange pay for their old age benefit. They note the importance of educational investment in improving labor productivity which helps sustain the social security program in the future. Richman and Stagner (1986) also claim that the existence of PAYG pension incentivizes investment in the younger cohorts by the older generation. Pogue and Sgontz (1977) also had a similar idea, informally argued in the context of social security taxation.

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[1] Government spending accounts for 91% of funds at primary, secondary, and post-secondary levels and 70% at the tertiary level in OECD countries. Public education spending in the United States accounts for 4.2% of GDP and 11.8% of total public spending (OECD, 2017a).

[2] OECD (2017b) report on public pensions states that 'Public pensions are often the largest single item of social expenditure, accounting for 18% of total government spending on average in 2013'. The old-age support expenditure, as a percentage of GDP in OECD countries, stands at 8.6%. Similarly, according to the report on education (OECD, 2017a) the education spending as a percentage of total government spending is 11.3% and as a percentage of GDP is at 4.4%.

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According to Rangel (2003), backward intergenerational goods, such as social security, play a crucial role in sustaining investment in the forward intergenerational goods like education.

In a seminal contribution, Boldrin and Montes (2005) formally show that one arm cannot sustain without the other.[3] Any increase in education subsidy necessarily hurts the working population who pay for the increase but do not benefit from it. This conflict is mitigated by interlinking education subsidies to old age income transfers. The authors establish that, when pension is used to compensate agents who foot the bill for education subsidies, it can replicate the complete market allocation (CMA), the *laissez-faire* allocation corresponding to the equilibrium with perfect education loan market. That is, an *education-pension policy package* can act as a substitute for the missing education loan market.

Do the welfare improvements coming from completing the education market dominate the welfare loss from keeping pensions around?[4] The classic Aaron-Samuelson result suggests that the PAYG pension program is welfare reducing in a dynamically efficient economy (Aaron, 1966) and hence, is not warranted.[5] This also runs counter to the policy suggestion in Boldrin and Montes (2005) that calls for pensions forever. We are interested in this inherent trade-off pertaining to pensions in the education-pension package: pensions serve the important role of compensating the generations that finance the education subsidy, but they are also welfare reducing in a dynamically efficient economy. In this paper, we study this trade-off by explicitly characterizing the optimal education-pension policy of the government along the entire path.

The existing literature highlights only some piecemeal aspects of the entire journey of this education-pension package. This extensive literature is silent on the optimal policy path and focuses mainly only on particular steady states. The only exception is Andersen and Bhattacharya (2017) who raise the concern of an ever-existing PAYG pension policy as suggested by Boldrin and Montes (2005) and put emphasis on the path, although their interest lies in improving allocations when the market is already complete. An analysis of the *optimal* policy path that captures rich dynamics of interdependence between education and pension starting from incomplete education loan market is therefore completely absent in the literature. Our paper is an attempt to fill this gap in the literature. We study the full-blown optimization exercise of a utilitarian government that maximizes the welfare of all the generations including the ones that are yet to come. Starting from an *incomplete* education loan market, a natural starting point to analyze government intervention, we focus on the characterization of the entire policy path that is *optimal* for the government and, at the same time, is Pareto improving for all generations.

Interestingly, for this optimal path, any policy recommending the co-existence of education subsidy and pension appears to be an 'interim' equilibrium. The economy with optimal policies moves away from the CMA over time. Following the optimal policy path, the economy achieves an education level that is *higher* than the CMA level. Along the optimal path, pension initially rises and then falls. The rising pension path is observed until the economy reaches the complete-market level of education. Pension support is done away with gradually in finite time once the CMA level of education is achieved and, eventually, education subsidy remains as the only working instrument of the policy package.

Hence, we show that the optimal policy for the government is to complete an otherwise incomplete education loan market with the help of PAYG pensions and eventually, achieve the best allocation (Golden Rule). It should follow, in the process, the rise and fall of pensions (starting from the incomplete market), ensuring Pareto improvement for all generations. Andersen and Bhattacharya (2017) observe that a rise and fall of pensions[6] is desirable if the welfare state wants to secure education that is higher than the CMA level. In their model, pension rises to support the growing education subsidies starting from the CMA and is phased out eventually with the help of intergenerational human capital externalities.[7] We do not need to rely on any other external factors; market imperfection itself is enough to generate the fall of pension and its eventual phase-out.

In a dynamically efficient economy,[8] showing the existence of a policy where PAYG pensions are eventually phased out without violating the Pareto constraint is an important contribution in itself.[9] Our interest lies in a complete phase-out of PAYG pensions in the presence of differently directed intergenerational transfers, crucially relying on the interdependence between them and the imperfections in the education loan market.[10] Apart from the compensatory role they play, PAYG pensions have little justification in the long-run if the economy is dynamically efficient. Crucial for our ability to show phase-out without violating the Pareto Criterion is that we start from a realistic consumption level when the market is incomplete. Because of market imperfections, initial consumption in our setup is low, and a Pareto move towards the planners' desirable outcome of zero pensions is made possible. Eventually, PAYG pension is phased out and the welfare state guarantees a consumption level that is higher than the best possible allocation under the competitive framework (CMA).

[3] It has also been a recurrent theme in the growth and political economy literature, for example in Kaganovich and Zilcha (1999), Kaganovich and Meier (2012), Lancia and Russo (2016).

[4] Undoubtedly, government transfers to aged people are quite significant. For example, Congressional Budget Office (2013) in the U.S. estimates that net transfers, defined as federal government transfers minus taxes, to households headed by a person over the age of 65 averaged 13,900 (in USD) whereas the corresponding transfers for non-elderly households with and without children were -16,900 (in USD) and -15,800 (in USD) respectively. It also reports that 15 percent of households aged 65 and above received over 60 percent of the federal transfers (also see Poterba, 2014).

[5] Also see Samuelson (1975) and Blanchard and Fischer (1989).

[6] Also see Bommier et al. (2010).

[7] Presence of such externalities is debatable (see, for example, Ciccone and Peri (2006), Acemoglu and Angrist (2000), Lange and Topel (2004), Yamarik (2008), and Rudd (2000)).

[8] Abel et al. (1989) show that the U.S. and other OECD countries are dynamically efficient. Mankiw (1995) mentions: "...excessive capital accumulation is not a practical concern for policymakers. Actual economies appear to have less capital than the Golden Rule level." Barbie et al. (2004) presents a test criterion based on Zilcha (1991) and robust evidence that the U.S. economy is dynamically efficient. From a theoretical perspective too, phasing out pensions is challenging only when the economy is dynamically efficient. Thus, following the literature, we also assume dynamic efficiency throughout the analysis.

[9] The broad literature on reforms towards downsizing the public pension benefits is rich and it by and large concludes that it is generally impossible to compensate the first generation of pensioners for the loss incurred without making at least one later generation worse off than under PAYG. Different papers in this literature use different setups. For example, Fenge (1995) uses a setup where agents are not liquidity constrained and, since a move from a PAYG to an actuarially fair fully funded system has no behavioral impact, there is no case for Pareto improvement. Breyer (1989) also has the same result while Rangel (1997) and Kotlikoff (2002) reach the same conclusion in a somewhat different context (also see Friedrich and Straub, 1993; Miles, 1999; Sinn, 2000, among others.). For a detailed discussion on this issue see Lindbeck and Persson (2003) and for related issues see Barr and Diamond (2006). Our framework of education with market imperfections and the related mechanism that exploits the link between education and pension for a Pareto improvement, as explained above, are substantially different from the existing literature.

[10] The literature also finds that the general equilibrium effects (see, for example, Cooley and Soares (1999) and Boldrin and Rustichini (2000)) or some socio-political reasons (see Boldrin and Rustichini (2000) and Bishnu and Wang (2017)) are crucial in sustaining a social security system. In our analysis we refrain from these issues and focus only on the efficiency angle.

Let us now briefly explain the mechanism at work in our paper. While market imperfections can arise for various reasons,[11] for tractability we model it in a simple way where imperfections create a wedge between the cost of borrowing for education and the return on savings. A government equipped with an education-pension policy package can access funds for education at the market rate of return by taxing today and returning the capitalized amount to the taxpayers in the future. Effectively, the government 'borrows' on behalf of the agents.[12] This difference in costs makes publicly financed education cheaper to the extent that distortions created by taxation are lower than those in the credit market. This, along with the assumption that publicly and privately funded education are good enough substitutes in the production function for human capital, makes the case for replacing private funding with public funding for economies afflicted by market imperfections.

A transition from private to public education without a system for compensating taxpayers necessarily hurts the initial working population who themselves did not benefit from the policy. This can be circumvented by providing old-age support to the agents who finance the public funding of education. In the literature, this is the idea behind pairing education subsidy with pension. However, in dynamically efficient economies, having PAYG pension in the steady-state is welfare reducing. This problem is mitigated in our paper in the following way. The government gradually expands the public system, increasing the tax burden on the working population. For the intervention to be a Pareto improvement, we require that every generation be at least as well off with the intervention than without. Pension is used to compensate the agents for their increased tax burden. Hence, pension payment and education subsidies go up until the marginal return from investment in education reaches the market rate of return (which is what holds at the CMA). Once the CMA level of education is achieved, increased aggregate resources (due to the increased level of education) can come to play the compensatory role of pensions. Build up of these 'extra' resources allow pensions to be phased out in finite time. Interestingly, these extra resources are generated within the system, that is, without the help from any external sources such as human capital externalities. Even after pension is phased out, the social cost of investment in education for the government through intergenerational transfers remains less than that for the agent under complete market. This difference in cost makes it optimal for the government to increase the investment in education beyond the complete market level. Eventually, the economy reaches the Golden Rule, the steady-state allocation that the social planner wants to achieve.

We complement our theoretical results with a section on numerical analysis. We observe that the optimal paths are clearly in line with our theoretical findings. The numerical exercise provides further insights into the model. Economies characterized by higher degrees of capital market imperfections take longer to reach the optimal level of education and consequently take longer to phase pensions out. On the other hand, governments that care more about future generations lead the economy to a higher level of education since the benefits of investment in education accrue to them. This also leads to a faster phase-out of pensions.

The rest of the paper is organized as follows. While Section 2 outlines the model, the *laissez-faire* equilibrium is described in Section 3. The government is introduced in Section 4, and Section 5 sets up the optimal policy intervention exercise by the government. We characterize the optimal public policies in Section 6. Section 7 contains the numerical version of our model. Section 8 concludes. All the proofs are presented in the Appendix.

2. The model

We consider an overlapping generations economy where agents live for three periods. They are young in the first period, middle-aged in the second, and old in the third. Time is discrete and indexed by $t = -1, 0, 1, 2, \ldots, \infty$. For simplicity, we assume that there is no population growth with the size of each generation being normalized to 1.

In our notation, a generation is identified by the period of their old-age. That is, we call an agent as belonging to generation t if she is old in period t. Thus a generation t agent is young in period $t-2$ and middle-aged in period $t-1$. In period $t-2$, young agents of generation t borrow an amount b_{t-2} in the credit market to invest in their education, e_{t-2}. The level of human capital h is realized after one period of investment in education and is assumed to be a strictly increasing and strictly concave function of the investment in education. That means e_{t-2} amount of investment made by a generation t agent in period $t-2$ generates human capital $h_{t-1} = h(e_{t-2})$ where $h'(.) > 0$ and $h''(.) < 0$.[13] Throughout the paper we assume that this human capital production function is free from any externalities such as spillovers from the parental level of education or the level of human capital of their cohorts in the economy. The factor prices are assumed to be exogenously given.[14] In the second period of life in $t-1$, agents supply labor inelastically, earning an exogenous wage rate w per unit of human capital. Once income is realized when they are middle-aged, agents repay their education loans taken when they were young.

For simplicity, we assume that agents consume only in the last period of their life. Agents save the entire net income s_{t-1} on which they earn an exogenous gross interest $R > 1$ when they are old.[15] Consumption of generation t agent who is old in period t is denoted by c_t. Since agents consume only in their old age, the utility of a generation t agent is given by $u(c_t)$. $u(.)$ is assumed to be strictly increasing, strictly concave and it follows Inada conditions, that is, $u'(.) > 0, u''(.) < 0$ with $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$. Agents are assumed to be non-altruistic; they maximize their own utility subject to the budget constraint.

The market for education loans is imperfect. While market imperfections can be modeled in several ways, for simplicity and tractability we consider that the credit markets are characterized by imperfections driving the cost of borrowing for education ρ above the market rate of return R, that is, $\rho > R$.[16] The borrowing cost ρ increases with the degree of imperfection, with sufficiently

[11] These market imperfections could arise from well-known sources of informational asymmetries such as moral hazard, adverse selection, imperfect enforcement, and so on, exacerbated, in general, because human capital cannot be pledged. See, for example, Friedman (1962), Nerlove (1975), Galor and Zeira, 1993; Stiglitz and Weiss, 1981 and Chapman (2006).

[12] We believe that the ability of the government to tax and therefore, effectively borrow from the middle-aged at the market rate of return is realistic and we do not arbitrarily use this: we insist on compensating all agents at every stage so that the taxation power cannot be deployed with full impunity.

[13] For completeness, we assume $wh'(0) > \rho$ (the borrowing rate of interest in the imperfect education loan market defined below) so that we avoid the corner solution $e = 0$.

[14] The importance of general equilibrium effects for sustaining intergenerational transfers is well known in the literature (see, for example, Cooley and Soares (1999), Boldrin and Rustichini (2000), Poutvaara (2004), Kothenburger and Poutvaara (2006)). While focusing on the efficiency angle, we want to ensure that our results are shown in the cleanest possible way, not influenced by the general equilibrium effects.

[15] We explore robustness to these assumptions in Section 7.3.

[16] These additional costs of borrowing can be justified in a setup where the lenders incur monitoring costs to ensure that borrowers do no run away as in Galor and Zeira (1993). Other market failures can also push the effective interest rate above the market interest rate (see, for example, Stiglitz and Weiss, 1981). This problem is even more severe in the market for education loans as, unlike physical capital, human capital is inalienable and cannot be mortgaged (see, for example, Friedman, 1962; Nerlove, 1975; Chapman, 2006).

large ρ representing the complete absence of any education loan market.

3. *Laissez-faire* equilibrium

To set up the benchmark, in this section we characterize the allocations first in the presence of credit market imperfections (incomplete markets) and then in its absence (complete markets).

3.1. Incomplete markets

Since there is no consumption in the first period and private borrowing is the only source of investment in education, total education expenditure equals private borrowing. An agent of generation t solves the following problem:

$$\begin{aligned}&\max_{b_{t-2},s_{t-1}} u(c_t),\\ &\text{subject to}\\ &0 \leqslant b_{t-2} \leqslant \tfrac{wh(e_{t-2})}{\rho},\\ &s_{t-1} + \rho b_{t-2} \leqslant wh(e_{t-2}),\\ &c_t \leqslant R s_{t-1},\\ &b_{t-2} = e_{t-2}.\end{aligned} \tag{1}$$

The first constraint, the no-default constraint, places an upper limit on the borrowings of the agent. The second and third constraints are the budget constraints for middle and old age respectively.

The solution to problem (1) is characterized by:

$$\begin{aligned} wh'(e^{IM}_{t-2}) &= \rho,\\ s^{IM}_{t-1} &= wh(e^{IM}_{t-2}) - \rho e^{IM}_{t-2}, \end{aligned}$$

where superscript *IM* represents the solution for incomplete market. Agents invest in education up to the point where the marginal benefit of education, $wh'(e^{IM}_{t-2})$, is equal to the marginal cost, ρ. Given our structure, agents do not value their second-period consumption and consume everything in the last period of life, $c^{IM}_t = R(wh(e^{IM}_{t-2}) - \rho e^{IM}_{t-2})$. This solution continues to hold period after period so that we drop the time subscript and define the incomplete markets allocation (e^{IM}, c^{IM}) by $wh'(e^{IM}) = \rho$, and $c^{IM} = R(wh(e^{IM}) - \rho e^{IM})$. In the following subsection we compare this allocation with the complete markets benchmark.

3.2. Complete markets

When the education loan market is complete and therefore there is no market failure, we have $\rho = R$. The allocation under this complete market is (e^{CM}, c^{CM}) that satisfies $wh'(e^{CM}) = R$ and $c^{CM} = R(wh(e^{CM}) - Re^{CM})$. This allocation is called the Complete Markets Allocation (CMA).

Under incomplete markets, imperfections prevent investment in education from reaching the complete markets level e^{CM} given by $wh'(e^{CM}) = R$, that is, $e^{IM} < e^{CM}$. Return on investment in education at the margin, $wh'(e^{IM})$, is strictly higher than the market rate of return R since $wh'(e^{IM}) = \rho > R$. Therefore a reallocation of resources towards investment in education can increase the total resource pie.

4. The government

In the presence of credit market imperfections, there is room for government intervention to improve allocations and welfare. The government is a welfare state with a utilitarian objective, that is, it maximizes the discounted sum of generational utilities, with the discount factor β reflecting social time preferences.[17]

$$W = \sum_{t=1}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1.$$

4.1. Dynamic efficiency and the Golden Rule

An assumption that is maintained throughout this paper is that our economy is dynamically efficient. Theoretically, the Golden Rule level of capital is the level that maximizes the sum of utilities of all the generations at the steady-state with equal weights assigned to all the generations. For any economy with a higher level of capital, there exists a Pareto improvement which reduces the capital stock, and such economies are termed as 'dynamically inefficient'. For economies with lower levels of capital, no such Pareto improvement is possible and these economies are called 'dynamically efficient'. In line with the other studies in this literature (as mentioned in the introduction), we also assume that our economy is dynamically efficient.

When utilities of future generations are discounted, the notion of 'Golden Rule' accordingly is changed to that of 'modified Golden Rule', which is the level of capital that maximizes the discounted sum of utilities. The modified Golden Rule approaches Golden Rule, in the limit, as the weight on future generations converges to one ($\lim \beta \to 1$). In an economy with production when there is no population growth (as in ours), it can be shown that the 'modified Golden Rule' level of capital is the one where the rate of return on capital (R) is the same as the inverse of generational discount factor ($1/\beta$). In the under-accumulation (dynamically efficient) region that we are presently focusing on, the rate of interest is higher than the inverse of discount factor ($R > 1/\beta$). For our analysis we assume that the same condition holds for the interest rate R. The other condition that needs to be satisfied at the modified Golden Rule is $wh'(e^{GR}) = 1/\beta$, where e^{GR} denotes investment in education at the modified Golden Rule.[18]

4.2. Achieving CMA through public policies

Market imperfections prevent consumption and investment in education from reaching the CMA level calling for a reallocation of resources towards education. In what follows we investigate whether state interventions can improve allocations and welfare. The government can make intergenerational transfers using lump-sum taxes and subsidies. In particular, the government can tax the middle-aged, and use lump-sum transfers to subsidize education for the young and provide pension to the old. The government has no other expenditure or sources of income. The government commits to the policy path announced.

In this paper, we focus on the most common form of government intervention in the education market – the provision of public education funded through taxation. Other policy instrument are available and used by governments, including subsidizing education loans for private education and funding public education by issuing debt. Optimal policy when allowing for these instruments

[17] A utilitarian social welfare function is not the only choice to represent a welfare state. A Rawlsian one, for example, can represent it very well. However, this utilitarian approach of deriving the optimal public policies by maximizing the sum of discounted generational utilities is very standard in the literature, especially in our intergenerational context of homogeneity within a particular generation but heterogeneity across generations. For example, Docquier et al. (2007) and Bishnu (2013) do so recently in the context of an economy with intergenerational education and pension transfers.

[18] It is straightforward to show that a planner who maximizes welfare through allocating resources would like to choose an allocation such that $R = 1/\beta$ along with $wh'(e^{GR}) = 1/\beta$ holds.

might be different from what we characterize, but we leave it for future work. Also note that if the government is allowed to borrow at the rate R, there may not be any need for formal pensions since, in general, debt and pensions have similar properties concerning intergenerational transfers.

We first explore whether and how the CMA could be achieved through public policies. Let g_t, p_t, and τ_t be the education subsidy, PAYG pensions and taxes for period t respectively. The government balances its budget in every period, hence

$$g_t + p_t = \tau_t \quad \forall t.$$

Total investment in education of generation t agent who receives g_{t-2} as education subsidies and borrows b_{t-2} in the credit market is given by $e_{t-2} = g_{t-2} + b_{t-2}$. Since public and private education expenditures are perfect substitutes, entering additively in the total education spending by the agent, public education crowds out private education one for one. It follows that when $g_t \geqslant e^{IM}$, we have $b_t = 0$ and $e_t = g_t$ for all $t \geqslant 1$.

In the presence of these fiscal instruments, a generation t agent's optimization problem gets modified as follows:

$$\begin{aligned}
&\max_{b_{t-2}, s_{t-1}} u(c_t), \\
&\text{subject to} \\
&0 \leqslant b_{t-2} \leqslant \frac{wh(e_{t-2})}{\rho}, \\
&s_{t-1} + \rho b_{t-2} + \tau_{t-1} \leqslant wh(e_{t-2}), \\
&c_t \leqslant R s_{t-1} + p_t, \\
&e_{t-2} = b_{t-2} + g_{t-2}.
\end{aligned} \tag{2}$$

The argument in the human capital production function is total education expenditure which is the sum of education subsidies and private borrowing. The modified budget constraint for the middle-aged reflects that the burden of the total tax is borne by them. On the other hand, the old age budget constraint captures the additional source of income in the form of pension.

With these fiscal instruments, we first show that a one-arm policy is not implementable. A balance between the two policy arms is needed to achieve the CMA.

Proposition 1.

(a) *A policy of providing only education subsidy to achieve the CMA necessarily hurts the initial middle-aged who at present pay the education tax for the future generation but did not receive any subsidy for their education.*
(b) *A policy package consisting of education and pension can achieve the CMA: there exists a sequence of education subsidy, PAYG pensions and taxes $\{g_t, p_t, \tau_t\}_{t=1}^{\infty}$, which implements the CMA without hurting any generation.*

Proof. See Appendix A. □

These results are similar to those proven in Boldrin and Montes (2005). A policy of only education subsidy necessarily hurts the initial middle-aged who pay the education tax but do not receive any subsidy in return. To compensate them for this loss, we require some pensions to be paid to them in their old age. In Appendix A we show that when government implements the policy package where $g_t = e^{CM}, p_t = Re^{CM}$, and $\tau_t = e^{CM} + Re^{CM}$ for all t, the agents optimally choose $b_{t-2} = 0$ and $s_{t-1} = wh(e^{CM}) - e^{CM} - Re^{CM}$ so that the resulting allocation is (e^{CM}, c^{CM}), the CMA. This demonstrates that an education subsidy should be accompanied by a strictly positive pension benefit to achieve the CMA.

5. Optimal public policies

Observe that the policy package discussed above has positive PAYG pensions along with education subsidies for all t to achieve the CMA. However, it has been shown theoretically that in dynamically efficient economies, a positive PAYG pension is welfare reducing (Aaron, 1966). Our interest lies in exploring this inherent trade-off involving pensions in the education-pension policy package. More importantly, what is missing in the literature is how to improve the allocations under the incomplete market in *an optimal public policy exercise of the government*. In what follows, we carry out this optimization exercise of the utilitarian government and investigate whether the resulting optimal public policies can complete the incomplete education loan market with the help of PAYG pensions and eventually lead the economy to the Golden Rule. We also characterize the entire policy paths, both for education and pension.

Along an equilibrium path with government intervention, agents solve problem (2) taking the policy path $\{g_t, p_t, \tau_t\}_{t=1}^{\infty}$ as given. On the other hand, the utilitarian government solves the following optimization problem taking into account the agents' response to the policy instruments:

$$\begin{aligned}
&\max_{\{g_t, p_t, \tau_t\}_{t=1}^{\infty}} W = \sum_{t=1}^{\infty} \beta^t u(c_t), \\
&\text{subject to} \\
&\tau_t \leqslant wh(e_{t-1}) - \rho b_{t-1} \quad \forall t, \\
&g_t + p_t = \tau_t \quad \forall t, \\
&g_t \geqslant 0 \quad \forall t, \\
&p_t \geqslant 0 \quad \forall t.
\end{aligned} \tag{3}$$

The first constraint reflects that the maximum that can be taxed away from an individual is limited by the amount of resources available to her after repaying the education loan. The second one is budget balancing by the government. Combining these two we arrive at the following constraint

$$g_t + p_t \leqslant wh(e_{t-1}) - \rho b_{t-1}$$

which we refer to as the *resource constraint*. The third and fourth constraints are the non-negativity constraints on education subsidy and pensions respectively.

In addition, we impose the condition that the policy is Pareto improving for all generations, that is, the utility of every generation under this policy is at least as high as that in its absence. This is captured by the following constraint which we refer to as the *Pareto constraint*[19]:

$$c_t \geqslant c^{IM} \qquad \forall t.$$

Pension allows the initial generations, which contribute to the program but do not receive the benefit, to be compensated. However, if there is no Pareto constraint and the government places a sufficiently large weight on the utility of future generations, it may not fully compensate the initial generations. Thus, to ensure implementation, we allow the government to make only Pareto improvements.

For simplicity of exposition, we assume that in $t = 1$ resources in the economy are sufficient to allow the government to raise, through taxation, at least the incomplete market level of investment in education e^{IM}. We argue that this implies the government choosing $\{g_t, p_t, \tau_t\}_{t=1}^{\infty}$ such that $g_t \geqslant e^{IM}$ and the agents choosing $\{b_{t-2}, s_{t-1}\}_{t=1}^{\infty}$ such that $b_t = 0$ is an equilibrium. Note that our

[19] While we set a lenient Pareto constraint of benchmarking only c^{IM} as consumption, eventually a stringent requirement has been satisfied where the consumption path is ever increasing.

results go through even if we relax this assumption. In such a situation the government will provide as much education as the resources allow for, and the agents borrow the rest to reach the incomplete market level of education. As public education is cheaper, this increases the resource pool in the economy in the next period and the process continues until public education moves beyond the incomplete market level and fully replaces the private education market.

Given $g_t \geqslant e^{IM}$, the solution to the agent's problem is:

$$b_t = 0 \quad \forall t \geqslant 1, \tag{4}$$

$$\begin{aligned} &c_1 = c^{IM} + p_1, \quad c_2 = c^{IM} + (p_2 - R(e_1 + p_1)), \\ &c_t = R(wh(e_{t-2}) - e_{t-1} - p_{t-1}) + p_t \quad \forall t \geqslant 3. \end{aligned} \tag{5}$$

The optimization problem of the government is to maximize $W = \sum_{t=1}^{\infty} \beta^t u(c_t)$ subject to the agent's optimal choice, the *resource constraint*, the *Pareto constraint*, and the non-negativity constraints on education subsidy and pensions. Since $b_t = 0$ for all $t \geqslant 1$, we have $g_t = e_t$ for all $t \geqslant 1$, so that the resource constraint becomes $e_t + p_t \leqslant wh(e_{t-1})$ for all $t \geqslant 2$. Note that, for $t = 1$, the resource constraint remains $e_1 + p_1 \leqslant wh(e^{IM}) - \rho e^{IM}$ since e_0 (and hence b_0) were already chosen at the incomplete market level e^{IM} before the initiation of government intervention in period $t = 1$. Substituting (5) into the utility function and the Pareto constraint, the government's optimization problem reduces to:

$$\begin{aligned} \max_{\{e_t,p_t\}_{t=1}^{\infty}} \; & u(c^{IM} + p_1) + \beta u(c^{IM} + p_2 - R(e_1 + p_1)) \\ & + \sum_{t=3}^{\infty} \beta^{t-1} u(R(wh(e_{t-2}) - e_{t-1} - p_{t-1}) + p_t) \\ \text{subject to} \; & \\ & e_1 + p_1 \leqslant wh(e^{IM}) - \rho e^{IM}, \quad e_t + p_t \leqslant wh(e_{t-1}) \quad \forall t \geqslant 2, \\ & c^{IM} + p_1 \geqslant c^{IM}, \quad c^{IM} + (p_2 - R(e_1 + p_1)) \geqslant c^{IM}, \\ & R(wh(e_{t-2}) - e_{t-1} - p_{t-1}) + p_t \geqslant c^{IM}, \quad \forall t \geqslant 3, \\ & e_t \geqslant 0 \quad \forall t \geqslant 1, \\ & p_t \geqslant 0 \quad \forall t \geqslant 1. \end{aligned} \tag{6}$$

Since $g_t = e_t$ and $\tau_t = g_t + p_t = e_t + p_t$, in problem (6) we rewrite the government's choices as $\{e_t, p_t\}_{t=1}^{\infty}$ instead of $\{g_t, p_t, \tau_t\}_{t=1}^{\infty}$ as in problem (3). This is consistent with the phrase *education-pension policy package* that we are using throughout the paper. The optimal path that we characterize below has $g_t = e_t \geqslant e^{IM} \quad \forall t \geqslant 1$. Thus, $\{g_t, p_t, \tau_t\}_{t=1}^{\infty}$ and $\{b_{t-2}, s_{t-1}\}_{t=1}^{\infty}$ such that $b_t = 0$ and $g_t \geqslant e^{IM} \quad \forall t \geqslant 1$ is indeed an equilibrium.

5.1. A feasible path

In this subsection, we show that the constraint set for problem (6) is non-empty. In particular, we show that there exists a feasible policy path which eventually reaches a steady state characterized by zero pensions and CMA level of investment in education e^{CM}. Along this steady state agents consume $R(wh(e^{CM}) - e^{CM}) > c^{CM} > c^{IM}$.

Proposition 2. *There exists a sequence $\{g_t, p_t, \tau_t\}_{t=1}^{\infty}$ which satisfies the constraints of problem (6), and eventually reaches a steady state with CMA level of investment in education and zero-pension.*

Proof. See Appendix B. □

The feasible path shown in Appendix B is described as follows. Till we reach the steady-state, we restrict the consumption of each generation to the minimum possible level c^{IM}. In the *laissez-faire* equilibrium, the agents were investing in education at a cost of $\rho > R$. The cost to the government for every unit of tax raised is only R (assuming it compensates the taxed agents via pensions). This difference in costs allows the total resources in the economy to increase. This gain in resources is not passed on to the agents and the government keeps on reinvesting it till the CMA level of investment in education is achieved. Once it is achieved, the increased resources are used to phase out pensions. Once the steady-state with zero pensions and CMA level of investment in education is reached, agents are allowed to have consumption strictly higher than c^{IM}.

This feasible path also allows us to rule out some other paths from being optimal. Along this path, the initial few generations are kept at c^{IM} while generations along the steady-state enjoy consumption strictly above c^{IM}. Any path along which all generations are at c^{IM} throughout is clearly dominated by this path and hence cannot be optimal. We use this observation later to rule out any such path from being optimal.

5.2. Necessary conditions for optimality

In Appendix C we show that an optimal policy path that solves problem (6) must satisfy the following first order conditions for all t:

$$-\lambda_t + \psi_t - \beta R[u'(c_{t+1}) + \eta_{t+1}] + \beta^2 Rwh'(e_t)[u'(c_{t+2}) + \eta_{t+2}] + \beta \lambda_{t+1} wh'(e_t) = 0,$$

$$u'(c_t) - \beta R u'(c_{t+1}) + (\eta_t - \beta R \eta_{t+1}) + \phi_t - \lambda_t = 0. \tag{8}$$

Here $\lambda_t, \eta_t, \psi_t$ and ϕ_t are the non-negative Lagrange multipliers respectively for the resource constraint, Pareto constraint, and non-negativity constraints on education subsidy and pension. Eqs. (7) and (8) are the first order conditions with respect to education subsidy and pension respectively.

Condition (7) captures the trade-off between consumption and investment in education. If we ignore the multipliers for the moment, the condition boils down to $u'(c_{t+1}) = \beta wh'(e_t) u'(c_{t+2})$. Leaving a unit of consumption with the generation $t+1$ agent yields a marginal utility $u'(c_{t+1})$, while investing it in the education of generation $t+2$ agent yields $wh'(e_t)u'(c_{t+2})$, which the planner discounts at the rate β. At the optimum, these two should be equal.

Condition (8) is the standard Euler equation. If we ignore the multipliers as before, the condition becomes $u'(c_t) = \beta R u'(c_{t+1})$. Giving a unit of consumption to generation t agent in the form of pension yields them a marginal utility $u'(c_t)$, while leaving it with generation $t+1$, allowing it to be saved, yields $Ru'(c_{t+1})$, which the planner discounts at the rate β. At the optimum, these two should be the same.

However, in general, all the Lagrange multipliers need not be zero at the same time. In what follows, we proceed to characterize optimal public policies as a solution to problem (6) under all possible scenarios.

6. Characterizing optimal public policies

In this section we characterize, step by step, the entire transition dynamics of the optimal education-pension policy package starting from an incomplete education loan market. The optimal public policies result as a solution to the welfare maximization exercise (problem (6)) of the utilitarian government. Before we get into the details of characterization, here is a prelude to the analysis and results through simple economic intuition. The characterization is established through some results that we develop as separate lemmas in Appendix D.

Throughout the analysis, the *private* education loan market remains incomplete. The benchmark consumption level in the Pareto constraint is set as the consumption under this incomplete market. This requirement is a bare minimum and easy to achieve, but the optimal path in fact satisfies a much harder constraint where consumption in each period is higher than the consumption in the previous period (see Lemma 1, Appendix D.1). We start implementing the optimal education-pension package from a period when the loan market is incomplete, called period $t = 1$ in our exercise.

We find that the optimal education-pension package can be characterized in three phases separated by periods T^* and $T^* + S$. T^* is the time period when the optimal education subsidy reaches the CMA level of investment in education; $e_t < e^{CM}$ for all $t < T^*$, and $e_t \geqslant e^{CM}$ for all $t \geqslant T^*$. On the other hand, $T^* + S$ is the time period when optimal pension becomes zero for the first time; the optimal pension is positive before $T^* + S$, and remains zero from $T^* + S$ onwards. In the first phase (periods 1 to T^*), both education subsidy and pension rise, the former reaching e^{CM} in period T^*. In the second phase (periods T^* to $T^* + S$), while education subsidy remains constant at e^{CM}, pension keeps falling till it becomes zero in period $T^* + S$. Finally, in the third phase (period $T^* + S$ onwards), pension remains at zero while education subsidy keeps increasing till it reaches the 'modified Golden Rule' level of investment in education in the limit.

Since investment in education is lower prior to the implementation of optimal policies, the government increases education subsidy to increase resources of the economy. But an increase in education subsidy hurts the initial middle-aged generation who pays for the increase but do not benefit from it. As compensation, the government needs to pay this generation a pension, which then requires it to pay pensions to the next generation also, and so on. Till the CMA level of education is achieved in period T^*, government uses all the resources available in the economy making the resource constraint bind throughout this period. This policy leaves middle-aged agents without any savings making them rely only on pension for consumption. A benevolent government in a dynamically efficient economy also increases (weakly) consumption over generations. To generate an increasing consumption path, the only option the government has is to choose an increasing pension path. Thus both components of the education-pension package rise till T^*. Once e^{CM} is achieved, the economy generates enough resources such that the resource constraint stops binding and agents start saving. With the help of these savings, dependency on the pension component of the package starts to weaken. The pension arm is completely phased out in period $T^* + S$. When both the instruments of intergenerational transfer were available, the government was equating the social returns on the two. Now that the pensions hit the zero lower bound, education remains the only working instrument. The social cost of investing in education is less than the private cost for the agent. Thus investment in public education starts increasing further from the CMA level, but this time it happens without any support from pension. The welfare state with only an education arm then eventually reaches the modified Golden Rule level, the steady-state that a social planner wants to achieve.

Our analysis confirms that a properly designed education-pension package not only completes the otherwise incomplete education loan market, it also leads the economy to the best possible allocation where pension is completely phased out. Interestingly, while the pension program is instrumental in the process of completing the education loan market, it is phased out once that objective is achieved. The optimal pension pattern follows rise and fall. We characterize these results formally in the next few subsections.

6.1. Completing the incomplete education loan market

We first establish that optimal public policies complete the incomplete education loan market with the help of PAYG pensions.

Proposition 3.

(a) *There exists some T such that* $e_T \geqslant e^{CM}$.
(b) *Suppose* T^* *is the first such period when* $e_t \geqslant e^{CM}$. *Then* $e_{t-1} \leqslant e_t$ *for all* $t \leqslant T^*$, *that is, education subsidy rises till period* T^*.

Proof. See Appendix E. □

Suppose $e_t < e^{CM}$ for all t. We first argue that this implies that the resource constraint binds for all t. This is established by showing that there exists a profitable deviation to the policy where $e_t < e^{CM}$ in some t for which the resource constraint does not bind. Non-binding resource constraint implies positive savings for generation $t + 1$. An additional unit of e_t will increase the earnings of generation $t + 2$ by $wh'(e_t)$ which can be transferred to generation $t + 1$ in the form of pension. The additional unit of tax decreases the earnings of generation $t + 1$ (through foregone savings) by R. As $wh'(e_t) > R$, this implies that this deviation increases the consumption of generation $t + 1$ agent without affecting any other generation.

Since the resource constraint binds in all periods, it leaves pensions as the only source of consumption for the agents. The government has to guarantee consumption of at least c^{IM} for each agent implying that it will have to give positive pensions in all periods. But we rule this out by the result that there exists at least one time period with zero pension (see Lemma 4, Appendix D.4).

In the following subsection, we show that pension rises till period T^*.[20] Since both education subsidy and pension tax increase from $T^* - 1$ to T^*, total resources of the economy must have increased between these two periods, implying a rise in education subsidy from $T^* - 2$ to $T^* - 1$. By a recursive argument, education subsidy increases monotonically till T^*.

In Proposition 5 below, we show that once the investment in education achieves the complete market level (e^{CM}), it does not fall below that level. Thus the optimal sequence of education subsidy, PAYG pensions and taxes completes the otherwise incomplete education loan market. Investment in the education level of the economy rises monotonically throughout this process.

6.2. Rise of pensions

Next, we show that the government keeps on increasing pensions to support this rise in education subsidies.

Proposition 4. *Suppose* T^* *is as defined in* Proposition 3. *Then* $p_{t-1} \leqslant p_t$ *for all* $t \leqslant T^*$, *that is, along with education subsidy, pension also rises till period* T^*.

Proof. See Appendix F. □

[20] It should be noted from the proof of Proposition 4 in Appendix F that the argument for the rise of pensions does not depend on the rise of education subsidies, that is, there is no circularity in our argument.

Proposition 3(a) shows that there exists some T in which education is above e^{CM}. T^* is the first such T. Consider all the periods preceding T^*. Education is below e^{CM} and, by the argument given above, we know that resource constraints bind in all these periods. Let us consider two consecutive generations $t-1$ and t. From the expression of consumption $(c_t = R(wh(e_{t-2}) - e_{t-1} - p_{t-1}) + p_t)$, note that if the resource constraint binds for period $t-1$ then the only source of consumption for generation t is the pensions they receive (p_t). So if the resource constraints bind in periods $t-2$ and $t-1$, then consumption is the same as pension for generation $t-1$ and generation t. But a benevolent government in a dynamically efficient economy increases (weakly) consumption over generations (see Lemma 1, Appendix D.1). It follows that $p_t = c_t \geqslant c_{t-1} = p_{t-1}$, that is, pensions rise in this period.

6.3. Fall and phase out of pensions

Once the complete market level of investment in education e^{CM} is achieved, the government starts phasing out pension. In the following proposition, we show that the pension falls (strictly) after T^*+1 and reaches zero in finite time. During this period, education subsidy stays constant at the CMA level. Pension is phased out completely in finite time, that is, once it falls to zero it stays there forever.

Proposition 5.

(a) *After T^*+1, pensions strictly fall till they become 0. During this period, education subsidy stays constant at e^{CM}.*
(b) *Suppose pension falls to zero in period T^*+S. Then for all $t \geqslant T^*+S$, the optimal solution has $p_t = 0$. That is, pension program is completely phased out from period T^*+S onwards.*

Proof. See Appendix G. □

After reaching e^{CM}, there are enough resources in the economy for the government to keep education subsidy weakly above e^{CM}. However, the following reasoning shows that the government is restrained from increasing education subsidy beyond e^{CM} as long as pension remains positive. When $p_{t+1} > 0$ and $e_t > e^{CM}$, reducing education subsidy to the young in t by one unit and giving a relief to the middle-aged in the form of reduced taxes will decrease the earning of the young of t by $wh'(e_t)$ but increase the earnings through savings of the middle-aged by R. This income gain can in turn be transferred to the young by reducing their pension tax in the next period as $p_{t+1} > 0$. Keeping e_{t+1} unchanged, this deviation leads to a strict increase in the net earnings of the generation $t+2$ agents without affecting any other generation. Thus education subsidy stays the same.

The optimal solution requires consumption to be increasing throughout. With constant investment in education, the only way to do so is via pension. Suppose the government increases pension for a generation at the expense of the next generation. In order to compensate the latter, a further increase in pension is required. This snowballs into an ever-increasing pension burden, making the path explode. The only way left is to reduce pension burden successively for each generation. Reducing pension burden is feasible after T^*+1 as generation T^*+1 is the first generation that makes positive savings in their middle age (in period T^*). Since agents now rely on both their pensions as well as positive savings for old age consumption, pensions need not track increasing consumption anymore. Pension will be phased out by increasing agents' reliance on their own savings. Savings keep on increasing as pensions are phased out since agents' earnings net of education tax remains the same with a constant investment in education.

We make an interesting observation in the context of completing the education loan market optimally. An education-pension package with positive pensions forever, as is often recommended in the literature (see Proposition 1), cannot be optimal. Rather it is 'interim' in nature: it completes the education loan market, but itself is not an optimal solution. As optimality requires in a dynamically efficient economy, pension is phased out once e^{CM} is achieved, without hurting any generation.

6.4. The long run and the golden rule

Finally, we examine where the economy reaches in the long run. In the next proposition, we show that once pension is phased out in finite time, investment in education under the welfare state eventually reaches the Golden Rule.

Proposition 6. *Suppose pension falls to zero in period T^*+S. Then for all $t \geqslant T^*+S$, the optimal solution has $e^{CM} \leqslant e_{t-1} < e_t < e^{GR}$ where $wh'(e^{GR}) = 1/\beta$. Moreover, $\lim_{t\to\infty} e_t = e^{GR}$.*

Proof. See Appendix H. □

The government has two instruments to mediate transfers between two subsequent generations – increasing education support and reducing pension. A unit taken from the previous generation has a social cost of $u'(c_t)$ while it yields a social benefit of $\beta wh'(e_{t-1})u'(c_{t+1})$ if invested in education and $\beta Ru'(c_{t+1})$ if used to reduce pension burden. This can be seen by rewriting the first order conditions for periods where resource, Pareto and non-negativity constraint on education do not bind:

$$\beta wh'(e_{t-1})u'(c_{t+1}) = u'(c_t) = \beta Ru'(c_{t+1}) + \phi_t, \tag{9}$$

where the first and second equality follow from Eqs. (7) and (8) respectively.

Whenever pension is non-zero, the government equates the returns from the education and pension channels. This can be seen in above equation which, upon setting $\phi = 0$ yields $wh'(e_{t-1}) = R$. With diminishing returns in education, educational investment is kept constant at e^{CM}, and the government instead focuses on phasing pension out.

However, once pension hits the zero lower bound $(\phi_t > 0)$, the only way left to mediate the transfer is via education. The social cost of investment in education is less than the private cost and hence the government expands education so long as this expansion is welfare increasing. This can be seen by rearranging the first equality above as

$$\beta wh'(e_{t-1}) = \frac{u'(c_t)}{u'(c_{t+1})}.$$

As the economy approaches the steady-state along the optimal path, investment in education reaches what we call the modified Golden Rule (MGR) defined as the level where $wh'(e^{GR}) = 1/\beta$ holds. Thus, in this process, the increase in investment in education is limited by the MGR level e^{GR}.

Note that the modified Golden Rule (MGR) level of investment in education is achieved when $wh'(e^{GR}) = 1/\beta$ whereas the CMA level of investment in education e^{CM} satisfies $wh'(e^{CM}) = R$. At the MGR, the other condition that typically needs to be satisfied in the closed economy models is $R = 1/\beta$. However, in our set-up with fixed factor prices, this parametric condition does not hold. Since we assume $R > 1/\beta$ throughout the paper, by construction the economy can potentially be sufficiently close to the MGR but cannot touch it. Thus, after guaranteeing a sufficiently higher level of investment in education than e^{CM} and phasing out pension completely in finite time, we find that education in our economy reaches the MGR in the limit.

7. Numerical analysis

In this section, we take our model and simulate the optimal education and pension paths. We demonstrate that the simulated optimal paths follow the qualitative characterization in the previous sections. Additionally, numerical analysis allows us to conduct comparative statics on key parameters of interest.

7.1. Baseline model

For the baseline model we assume log utilities and Cobb-Douglas human capital function, i.e., $u(c) = \log(c)$ and $h(e) = e^{\alpha}$, with $\alpha = 0.5$. A generation lives for 25 years. The annual complete market interest rate is 1.25% and the annual incomplete market rate of interest is 3%. In the model this corresponds to an intergenerational complete market interest rate (R) of 1.36 and intergenerational incomplete market interest rate (ρ) of 2.09. Annual discount factor is 0.99 which corresponds to an intergenerational discount factor (β) of 0.78.

Fig. 1 plots the optimal education, pension, and income paths for our baseline model. Consumption follows the same path as income. The optimal path can be divided into three phases. Phase 1, from $t = 1$ till $t = T^*$, education keeps on rising till it reaches the complete market level while pensions also increase. In Phase 2, between T^* and $T^* + S$, the government keeps education at the complete market level while pensions are phased out. Finally, in Phase 3, starting $T^* + S$ government increases education to the golden rule level. Income (in 1(c)) strictly increases in all the phases.

7.2. Comparative statics

We explore two comparative statics exercises numerically. In Fig. 2 we plot the optimal paths for different degrees of credit market imperfection (ρ). As seen in Fig. 2(a), the starting level of education is lower with a higher level of credit market imperfection. Further, the convergence to the complete market level of education and the golden rule level of education is faster when the degree of imperfection is lower. Similarly, the optimal pension path peaks and goes to zero earlier for a lower degree of imperfection, as depicted in Fig. 2(b).

Fig. 3 plots the optimal education and pension paths for varying levels of discount factor (β). A government with higher β values future generations more and phases out pensions earlier, as seen in Fig. 3(b).

7.3. Two period consumption and imperfect consumption credit market

So far we have assumed that agents consume only in their old-age which, under the assumption of a perfect consumption credit market, yields a setup equivalent to the one with two-period consumption. While this assumption improved the analytical tractability of our results, we explore the robustness of our results to this assumption.

When there are no imperfections in the credit market for consumption, it is straightforward to allow consumption in the middle-age and the results remain unchanged, both quantitatively and qualitatively. With constant factor prices, maximizing utility is equivalent to maximizing lifetime income, which doesn't depend on whether consumption is allowed in middle age or not. Intuitively, if agents have access to perfect consumption credit market, maximizing old-age consumption under one-period consumption is equivalent to maximizing the present value of lifetime income

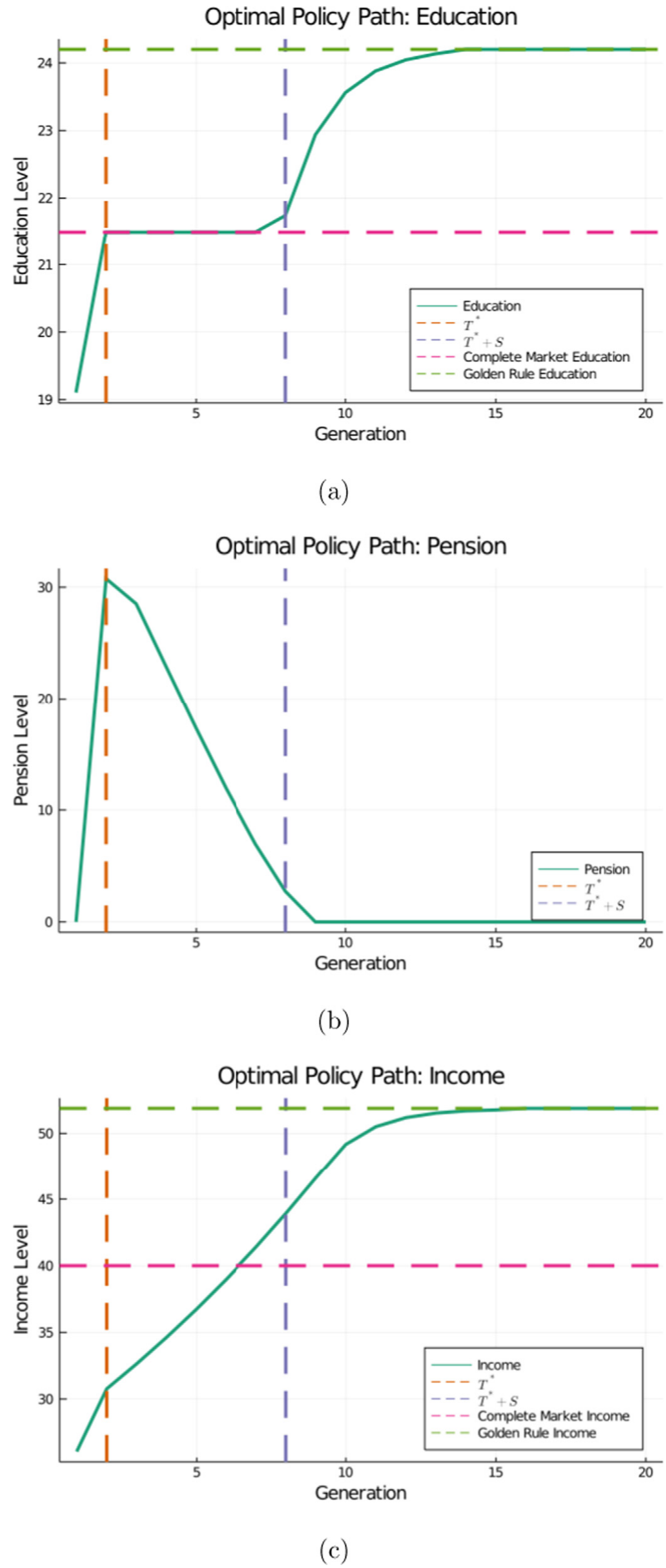
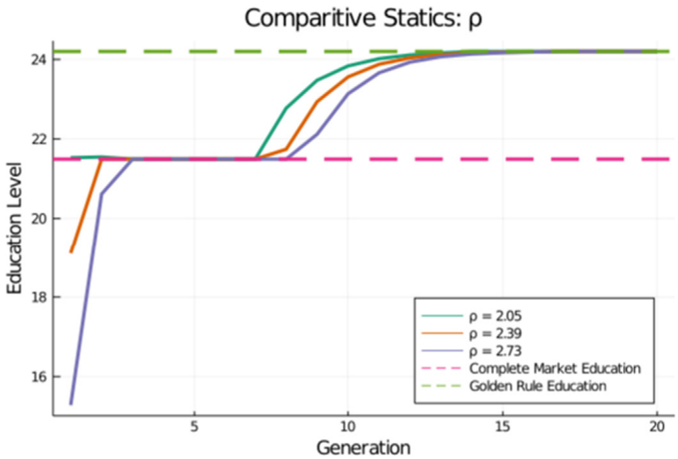
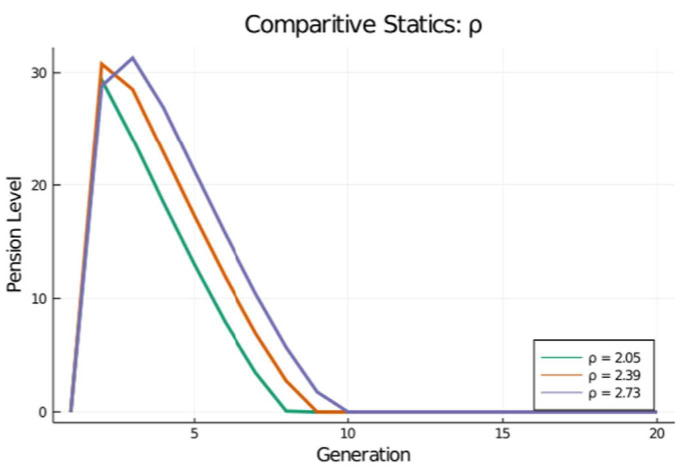


Fig. 1. Optimal Policy Paths. Panel (a) plots the optimal education path, Panel (b) plots the optimal pension path and Panel (c) plots the optimal income path.

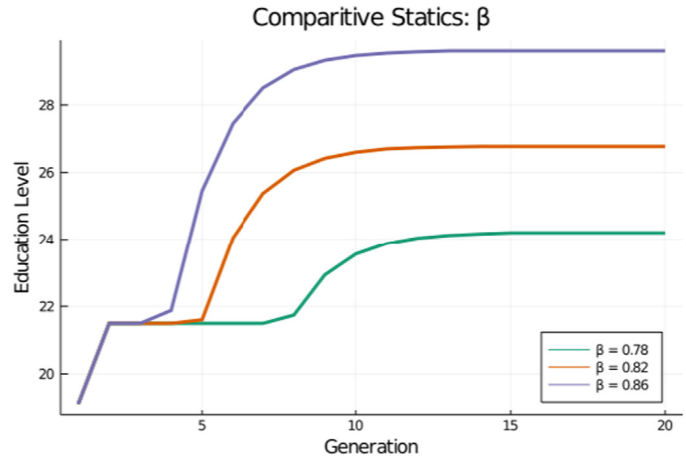


(a)

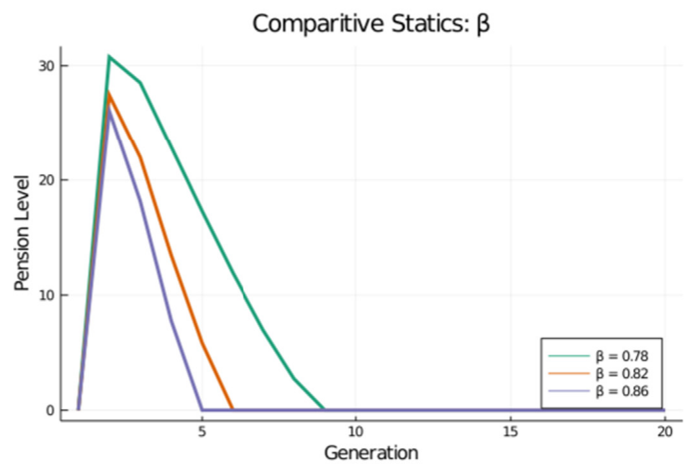


(b)

Fig. 2. Comparative Statics:ρ. Panel (a) plots the optimal education path, Panel (b) plots the optimal pension path.



(a)



(b)

Fig. 3. Comparative Statics: β. Panel (a) plots the optimal education path, Panel (b) plots the optimal pension path.

under two-period consumption. Agents' policy functions do not change and we get the same optimal policy path as before.

However, the assumption of a perfect consumption credit market might not be innocuous ex-ante. If agents face an imperfect credit market in the middle age then the government will have an incentive to moderate the middle-age tax to reduce the interest burden. Hence, we explore the robustness of our results to this assumption through the numerical version of the model.

The new lifetime utility function is assumed to be $u(c^m, c^o) = log(c^m) + \delta log(c^o)$, where c^m is the consumption in the middle age, c^o is the consumption when old and δ is the discount factor. We set δ to be the same as β. The results are reported in Fig. 4. When consumption credit market is allowed to be perfect, the optimal policy paths look the same as the baseline validating the intuition outlined in the previous paragraph. Next, we set the interest rate for borrowing during middle age to be the same as the one in the education loan market, ρ, while saving is still at the market rate of return R. The optimal policy paths take longer to reach the CMA and the maximum pension amount is lower. The government raises lower taxes and supports lower pension because taxing the middle age too much will force them to borrow, which now has a higher efficiency cost. Interestingly, the convergence to zero pension level is faster under an imperfect consumption credit market, likely because the amount of pension to be phased out now is lower.

8. Conclusion

A welfare state equipped with backward and forward intergenerational transfers emerges as a rescuer in economies where the education loan market is either primitive or missing. We show that it can do a lot more. A double-armed welfare state is capable of leading the economy not only to the steady-state corresponding to the perfect credit market (CMA) but also eventually to the one that the social planner finds to be the best (Golden Rule). This result has been shown in a full-blown optimization exercise of a benevolent government that maximizes the welfare of all the generations, while honoring the Pareto criterion and guaranteeing an improvement in welfare over generations.

Our framework is fairly general, yet tractable enough, to allow for analytical characterization of the optimal policy path. We have verified that our qualitative results are robust to any change in the

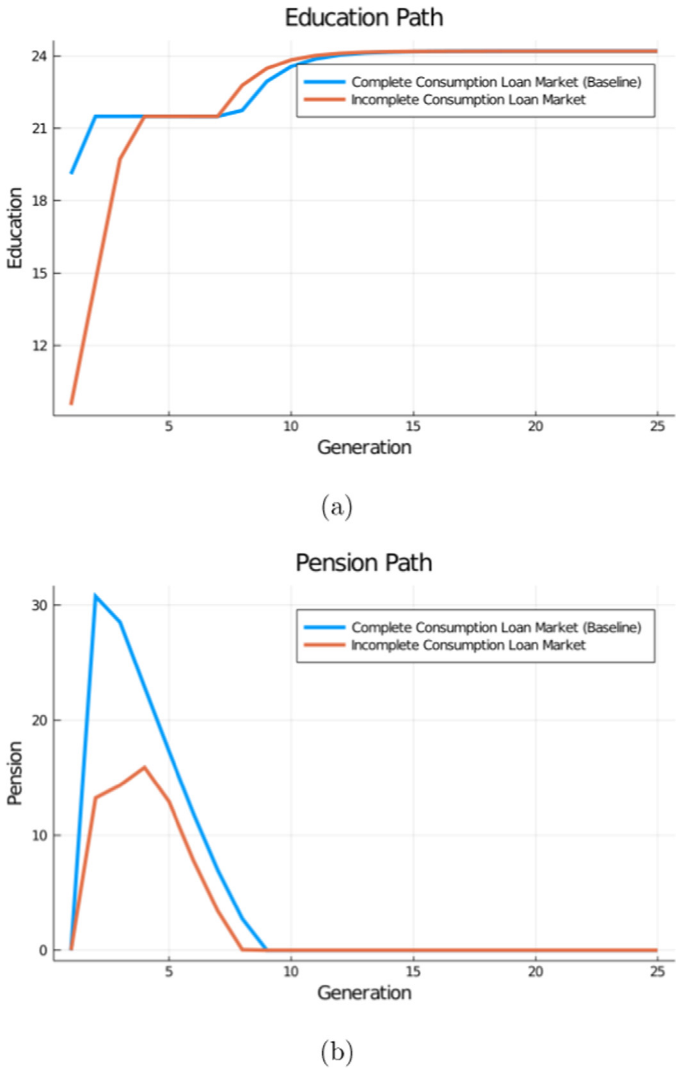


Fig. 4. Optimal Policy Paths under Two-Period Consumption. Panel (a) plots the optimal education path, Panel (b) plots the optimal pension path.

severity of the capital market imperfection or the discount rate of the government. Further, in an extension to a quantitative model, we allow for two-period consumption (in middle age and old) as well as imperfection in consumption credit markets. Higher borrowing costs in the middle age change the cost of raising taxes and therefore the optimal policy path, but the new path remains qualitatively similar. In another extension, we have also explored how the results change when parents are altruistic and derive utility directly from their children's level of education. Due to the direct utility channel, they need to be compensated less via pensions which results in faster convergence to the golden rule level of education.

An assumption we maintain throughout is that of a small open economy. When factor prices are endogenous, additional forces come into play. Increasing PAYG pensions reduces savings and hence the capital stock. This in turn rewards the old (higher return on their capital) at the expense of the middle-aged (lower wages). In addition to that, investment in education will also have effects through the factor prices. How all these effects will interact and affect the policy path is unclear *ex-ante*, although, we believe that the fundamental forces that we have outlined should push the path to be qualitatively similar. We leave this to future work.

The education-pension policy package that we propose is powerful enough to take the economy to the CMA, but once this CMA is achieved, pension component of the package can and should be phased out. Thus, in this analysis, the dual objective of completing the education loan market as well as phasing out PAYG pensions emerges as an optimal choice of a utilitarian government. The optimal pension path that results from the analysis follows a rise and fall of PAYG pensions. While some piecemeal analyses of the entire journey have appeared in the literature, the need for showing the entire path of the optimal policy including the transition dynamics has gone unnoticed. Our contribution lies precisely in filling up this void in the literature.

Appendix A. Proof of Proposition 1

A.1. Proof of Proposition 1(a)

Proof. Consider problem (2). We ignore the no-default constraint for the time being. The optimal solution obtained indeed satisfies the constraint. Let υ_1 and υ_2 be the Lagrange multipliers for the constraint $0 \leqslant b_{t-2}$ and for the middle age budget constraint, respectively. Since the agents are not altruistic, they consume everything when they are old so that the old age budget constraint holds with equality. We substitute the two equality constraints into the objective function of the agent.

The Langrangian is given by

$$\mathscr{L} = u(Rs_{t-1} + p_t) + \upsilon_1 b_{t-2} + \upsilon_2(wh(b_{t-2} + g_{t-2}) - s_{t-1} - \rho b_{t-2} - \tau_{t-1}).$$

Differentiating with respect to b_{t-2} and s_{t-1} we get

$$\frac{\partial \mathscr{L}}{\partial b_{t-2}} = \upsilon_1 - \upsilon_2(\rho - wh'(b_{t-2} + g_{t-2})),$$

$$\frac{\partial \mathscr{L}}{\partial s_{t-1}} = u'(c_t)R - \upsilon_2.$$

Setting both the equations equal to zero, we get the following first order conditions (along with the corresponding complementary slackness conditions):

$$\upsilon_2[\rho - wh'(b_{t-2} + g_{t-2})] = \upsilon_1,$$

$$u'(c_t)R = \upsilon_2.$$

Since $u'(c_t) > 0, \upsilon_2 > 0$, implying that the middle age budget constraint binds. That is, agents save their entire earnings net of tax payments and loan repayments.

Consider the case when the non-negativity constraint on borrowing binds, that is, $b_{t-2} = 0$. By complementary slackness, we have $\upsilon_1 \geqslant 0$. Since $\upsilon_2 > 0$, this implies that $\rho \geqslant wh'(g_{t-2})$. Thus, we have $g_{t-2} \geqslant e^{IM}$. It follows that whenever $g_{t-2} < e^{IM}$, we have $b_{t-2} > 0$. This implies that $\rho = wh'(b_{t-2} + g_{t-2})$, or $b_{t-2} = e^{IM} - g_{t-2}$.

Hence, given g_{t-2}, the optimal borrowing and savings is given by:

$$(b_{t-2}, s_{t-1}) = \begin{cases} (e^{IM} - g_{t-2}, wh(e^{IM}) - \rho(e^{IM} - g_{t-2}) - \tau_{t-1}) & \text{if } g_{t-2} < e^{IM} \\ (0, wh(g_{t-2}) - \tau_{t-1}) & \text{if } g_{t-2} \geqslant e^{IM}. \end{cases} \tag{A.1}$$

Consider the policy of providing only the education subsidy and no pensions starting from $t = 1$. That is, $g_t > 0$ and $p_t = 0$ for all $t \geqslant 1$. This education subsidy is financed by taxing the working population, i.e. $\tau_t = g_t > 0$.

Consider the agent of generation 2 who is middle aged in period 1. We call this the initial middle aged agent. Since $g_0 = 0$, from (A.1) we know that the agent responds to the policy by choosing $b_0 = e^{IM}$. Hence, his savings and consumption with this policy are given by $s_1 = wh(e^{IM}) - \rho e^{IM} - g_1$ and $c_2 = R(wh(e^{IM}) - \rho e^{IM} - g_1)$. Note that since $g_1 > 0$ this c_2 is strictly less than $c^{IM} = R(wh(e^{IM}) - \rho e^{IM})$. Thus, this policy necessarily hurts the middle aged. □

A.2. Proof of Proposition 1(b)

Proof. Consider the policy $g_t = e^{CM}, p_1 = 0, p_{t+1} = Re^{CM}, \tau_1 = e^{CM}$ and $\tau_{t+1} = e^{CM} + Re^{CM}$ for all $t \geqslant 1$. Since $g_{t-2} > e^{IM}$, the optimal choice of generation $t \geqslant 3$ (given by Eq. (A.1)) is $b_{t-2} = 0$ and $s_{t-1} = wh(e^{CM}) - e^{CM} - Re^{CM}$. Thus, for $t \geqslant 3, e_{t-2} = e^{CM}$ and the consumption becomes $c_t = R(wh(e^{CM}) - Re^{CM})$, which is as under complete markets. Note that $c_1 = c_2 = c^{IM}$ under this policy, so that the policy does not hurt any generation. □

Appendix B. Proof of Proposition 2

Proof. We show that there exists a feasible policy path which eventually reaches a steady state characterized by zero pensions and CMA level of investment in education e^{CM}. Along this path, till reaching the zero pension steady state, we keep every generation at the incomplete market level of consumption c^{IM}. We define the policy path period by period.

Period $1 : p_1 = 0$ and $e_1 = wh(e^{IM}) - \rho e^{IM} > e^{IM}$. Old agents in period 1 get no pension and stay at c^{IM}. The government taxes away all the income of middle-aged agents and uses it to fund education. Recall that, for ease of exposition, we have assumed that in period 1 resources available in the economy, $wh(e^{IM}) - \rho e^{IM}$, are sufficient to allow the government to raise at least e^{IM} in taxes.

Period $2 : p_2 = c^{IM}$ and $e_2 = wh(e_1) - c^{IM}$. We show that education strictly increases between periods 1 and 2. Note that the maximum of the function $wh(e) - Re$ is achieved at $wh'(e^{CM}) = R$ and, for all $e < e^{CM}$, the function is increasing. As $e_1 > e^{IM}$, we have

$$\begin{aligned}
& wh(e_1) - Re_1 > wh(e^{IM}) - Re^{IM} > wh(e^{IM}) - \rho e^{IM} = e_1 \\
\Rightarrow\ & wh(e_1) - c^{IM} > e_1 \qquad [\text{since } c^{IM} = R(wh(e^{IM}) - \rho e^{IM}) = Re_1] \\
\Rightarrow\ & e_2 > e_1.
\end{aligned}$$

Similarly, for any period $t > 1$ till education is below e^{CM}, the policy is defined as $p_t = c^{IM}$ and $e_t = wh(e_{t-1}) - c^{IM}$. Using $e_2 > e_1$ it is easy to argue inductively that $e_{t+1} > e_t$, that is, as long as education is below e^{CM}, it keeps on increasing.

Moreover education increases at an increasing rate:

$$\begin{aligned}
& wh(e_t) - wh(e_{t-1}) > wh'(e_t)(e_t - e_{t-1}) \qquad [\text{follows from concavity of } h(.)] \\
\Rightarrow\ & wh(e_t) - wh(e_{t-1}) > e_t - e_{t-1} \qquad [\text{since } wh'(e_t) > R > 1 \text{ as } e_t < e^{CM}] \\
\Rightarrow\ & (wh(e_t) - c^{IM}) - (wh(e_{t-1}) - c^{IM}) > e_t - e_{t-1} \\
\Rightarrow\ & e_{t+1} - e_t > e_t - e_{t-1}.
\end{aligned}$$

It follows that education reaches e^{CM} in finite number of periods. Suppose education reaches e^{CM} in some period T. From period $T+1$ onwards we keep e_t at e^{CM} and adjust pensions just enough to keep the consumption of previous generation at c^{IM}. We show that each generation will require less pension than the previous generation so that pensions can be phased out.

Since consumption is the sum of earning through savings and pensions, we define policies for periods $T+1$ and $T+2$ as follows.

Period T + 1: $p_{T+1} = c^{IM} - R(wh(e_{T-1}) - e^{CM} - c^{IM})$ and $e_{T+1} = e^{CM}$.

Period T + 2: $p_{T+2} = c^{IM} - R(wh(e^{CM}) - e^{CM} - p_{T+1})$ and $e_{T+2} = e^{CM}$.

We show that pension falls between periods $T+1$ and $T+2$:

$$\begin{aligned}
& wh(e_{T-1}) - e^{CM} - c^{IM} < wh(e^{CM}) - e^{CM} - c^{IM} + R(wh(e_{T-1}) - e^{CM} - c^{IM}) \\
& [\text{since } e_{T-1} < e^{CM} \text{ and } e^{CM} + c^{IM} \leqslant wh(e_{T-1}) \text{ by the resource constraint}] \\
\Rightarrow\ & c^{IM} - R(wh(e^{CM}) - e^{CM} - c^{IM} + R(wh(e_{T-1}) - e^{CM} - c^{IM})) < c^{IM} - R(wh(e_{T-1}) - e^{CM} - c^{IM}) \\
& \Rightarrow p_{T+2} < p_{T+1}.
\end{aligned}$$

Similarly, for any period $t > T+1$ the policy is defined as $p_t = c^{IM} - R(wh(e^{CM}) - e^{CM} - p_{t-1})$ and $e_t = e^{CM}$. Using $p_{T+2} < p_{T+1}$ it is easy to argue inductively that $p_{t+1} < p_t$, that is, pension keeps on falling.

Moreover pensions fall at an increasing rate. From the construction of pension in the falling pension region given above it follows that

$$\begin{aligned}
& p_{t+1} - p_t \\
& = (c^{IM} - R(wh(e^{CM}) - e^{CM} - p_t)) - (c^{IM} - R(wh(e^{CM}) - e^{CM} - p_{t-1})) \\
& = R(p_t - p_{t-1}) \\
& > p_t - p_{t-1}. \qquad [\text{since } R > 1]
\end{aligned}$$

Thus pensions fall and reach zero in finite time. After pension falls to zero, we keep pensions at zero and education at e^{CM} in all the following periods. Each generation consumes $R(wh(e^{CM}) - e^{CM}) > c^{IM}$ in the steady state.

Thus there exists a feasible policy path which reaches the CMA level of education and zero pension steady state. □

Appendix C. First order conditions for Problem (6)

Let $\lambda_t, \eta_t, \psi_t$ and ϕ_t be the non-negative Lagrange multipliers respectively for the resource constraint, Pareto constraint, and non-negativity constraints on education subsidy and pension.

The Lagrangian is given by

$$\begin{aligned}
\mathcal{L} = \ & u(c^{IM} + p_1) + \beta u(c^{IM} + p_2 - R(e_1 + p_1)) + \sum_{t=3}^{\infty} \beta^{t-1} \{u(R(wh(e_{t-2}) - e_{t-1} - p_{t-1}) + p_t)\} + \\
& \lambda_1 [wh(e^{IM}) - \rho e^{IM} - e_1 - p_1] + \sum_{t=2}^{\infty} \beta^{t-1} \{\lambda_t [wh(e_{t-1}) - e_t - p_t]\} + \\
& \eta_1 p_1 + \beta \eta_2 [p_2 - R(e_1 + p_1)] + \sum_{t=3}^{\infty} \beta^{t-1} \{\eta_t [R(wh(e_{t-2}) - e_{t-1} - p_{t-1}) + p_t - c^{IM}]\} + \\
& \sum_{t=1}^{\infty} \beta^{t-1} \{\psi_t e_t + \phi_t p_t\}.
\end{aligned}$$

Differentiating with respect to e_t and p_t gives us

$$\frac{\partial \mathcal{L}}{\partial e_t} = -\lambda_t + \psi_t - \beta R[u'(c_{t+1}) + \eta_{t+1}] + \beta^2 Rwh'(e_t)[u'(c_{t+2}) + \eta_{t+2}] + \beta \lambda_{t+1} wh'(e_t), \tag{C.1}$$

$$\frac{\partial \mathcal{L}}{\partial p_t} = u'(c_t) - \beta Ru'(c_{t+1}) + (\eta_t - \beta R\eta_{t+1}) + \phi_t - \lambda_t. \tag{C.2}$$

Setting (C.1) and (C.2) equal to 0, along with the complementary slackness conditions, gives us the first order conditions, Eqs. (7) and (8), in the text.

Appendix D. Four Lemmas and their Proofs

D.1. Lemma 1 (Increasing Consumption)

Lemma 1. *If $p_t > 0$, then $c_{t+1} \geqslant c_t$ for all t.*

Proof. As $p_t > 0, \phi_t = 0$ by complementary slackness condition. Then first order condition (8) becomes

$$u'(c_t) - \beta R u'(c_{t+1}) + (\eta_t - \beta R \eta_{t+1}) - \lambda_t = 0.$$

Note that $\eta_t \geqslant 0$ is the multiplier associated with the Pareto constraint $c_t \geqslant c^{IM}$. There are four cases to consider.

Case 1: $\eta_t > 0$ and $\eta_{t+1} > 0$. By complementary slackness both c_t and c_{t+1} are equal to c^{IM}, and hence $c_t = c_{t+1}$.

Case 2: $\eta_t > 0$ and $\eta_{t+1} = 0$. Then $c_t = c^{IM}$ and $c_{t+1} \geqslant c^{IM}$. Hence the required inequality holds.

Case 3: $\eta_t = 0$ and $\eta_{t+1} = 0$. From the first order condition we get

$$\begin{aligned} & u'(c_t) = \beta R u'(c_{t+1}) + \lambda_t \\ \Rightarrow\ & u'(c_t) > u'(c_{t+1}) \qquad [\text{since } \beta R > 1 \text{ and } \lambda_t \geqslant 0] \\ \Rightarrow\ & c_{t+1} > c_t. \end{aligned}$$

Case 4: $\eta_t = 0$ and $\eta_{t+1} > 0$. It follows from the first order condition that

$$\begin{aligned} & u'(c_t) = \beta R u'(c_{t+1}) + \lambda_t + \beta R \eta_{t+1} \\ \Rightarrow\ & u'(c_t) > u'(c_{t+1}) \qquad [\text{since } \beta R > 1, \lambda_t \geqslant 0 \text{ and } \eta_{t+1} > 0] \\ \Rightarrow\ & c_{t+1} > c_t. \end{aligned}$$

But since $\eta_{t+1} > 0, c_{t+1} = c^{IM}$ by complementary slackness condition. It follows that $c_t < c^{IM}$, a contradiction. Hence this case cannot arise.

Thus, in all the cases that can arise in the optimal solution, we have $c_{t+1} \geqslant c_t$ when $p_t > 0$. □

D.2. Lemma 2 (Rising Pensions)

Lemma 2. *If resource constraints bind in any two consecutive periods $t-1$ and t along with $p_t > 0$, then $p_{t+1} \geqslant p_t$.*

Proof. Since $p_t > 0$, from Lemma 1, we have $c_{t+1} \geqslant c_t$. Substituting the expressions for consumption in terms of education and pension we get

$$\begin{aligned} & c_{t+1} \geqslant c_t \\ \Rightarrow\ & R(wh(e_{t-1}) - e_t - p_t) + p_{t+1} \geqslant R(wh(e_{t-2}) - e_{t-1} - p_{t-1}) + p_t \\ \Rightarrow\ & p_{t+1} \geqslant p_t. \qquad [\text{since resource constraints bind}] \end{aligned}$$
□

D.3. Lemma 3 (Education Subsidies)

Lemma 3.

(a) *If resource constraint binds for period t, then $e_t \leqslant e^{CM}$.*
(b) *If resource constraint does not bind for period t, then $e_t \geqslant e^{CM}$. Additionally, if $p_{t+1} > 0$, then $e_t = e^{CM}$.*

Proof. Updating Eq. (8) by one period we get

$$-\lambda_{t+1} + u'(c_{t+1}) - \beta R u'(c_{t+2}) + (\eta_{t+1} - \beta R \eta_{t+2}) + \phi_{t+1}$$

Substituting this expression for $u'(c_{t+1})$ in Eq. (7), we get

$$-\lambda_t + \psi_t + \beta(wh'(e_t) - R)(\beta R u'(c_{t+2}) + \beta R \eta_{t+2} + \lambda_{t+1}) + \beta R \phi_{t+1} = 0.$$

Consider the case when the resource constraint binds in period t, that is, $e_t + p_t = wh(e_{t-1})$. This implies that $s_t = 0$ and $c_{t+1} = p_{t+1}$. To ensure positive consumption (due to Inada condition), pension in period $t+1$ must be positive. Hence, $\phi_{t+1} = 0$. Suppose, on the contrary, $e_t > e^{CM} (> 0)$. This implies that $\psi_t = 0$, and Eq. (D.1) becomes

$$\beta(wh'(e_t) - R)(\beta R u'(c_{t+2}) + \beta R \eta_{t+2} + \lambda_{t+1}) = \lambda_t.$$

Since the resource constraint binds, $\lambda_t \geqslant 0$. Since $\eta_{t+2} \geqslant 0, \lambda_{t+1} \geqslant 0$ and $u'(c_{t+2}) > 0$, we have $\beta R u'(c_{t+2}) + \beta R \eta_{t+2} + \lambda_{t+1} > 0$. This implies that $wh'(e_t) \geqslant R$, or $e_t \leqslant e^{CM}$ which gives us the contradiction.

Now consider when the resource constraint does not bind in period t. Substituting for $\lambda_t = 0$ in Eq. (D.1), we get

$$\psi_t + \beta(wh'(e_t) - R)(\beta R u'(c_{t+2}) + \beta R \eta_{t+2} + \lambda_{t+1}) + \beta R \phi_{t+1} = 0.$$

For the equality to hold, the second term must be non-positive (as $\phi_{t+1} \geqslant 0$ and $\psi_t \geqslant 0$). This in turn implies that $wh'(e_t) \leqslant R$ resulting in $e_t \geqslant e^{CM}$.

Since $e_t \geqslant e^{CM} > 0$, from complementary slackness, we get that $\psi_t = 0$. Additionally, if $p_{t+1} > 0$, then $\phi_{t+1} = 0$. Since $\beta R u'(c_{t+2}) + \beta R \eta_{t+2} + \lambda_{t+1} > 0$, for ϕ_{t+1} to be zero we must have $wh'(e_t) = R$, that is, $e_t = e^{CM}$. □

D.4. Lemma 4 (Zero Pension)

Lemma 4. *There exists a time period $Z \geqslant 2$ such that $p_Z = 0$.*

Proof. We prove this lemma by contradiction. Let us assume that $p_t > 0$ for all $t \geqslant 2$. We consider the following exhaustive cases and argue that a contradiction arises in each.

Case 1: Consumption stays at c^{IM} for all the generations. Discussion in Section 5.1 shows that there exists a feasible path which dominates this path and hence this path cannot be an optimal one.

Case 2: Consumption is strictly above c^{IM} for some t. There are two sub-cases to consider: Case 2(a): $p_1 > 0$ and Case 2(b): $p_1 = 0$.

Case 2(a): $p_1 > 0$. Since $c_1 = c^{IM} + p_1$ (see Eq. (5)), with $p_1 > 0, c_1 > c^{IM}$. As consumption is weakly rising, consumption stays above c^{IM} for all subsequent periods. Consider the first order condition (8). The multipliers associated with the Pareto constraint, η_t, drop out.[21] Then we manipulate condition (8) as follows:

$$\begin{aligned} & u'(c_t) = \beta R u'(c_{t+1}) + \lambda_t \\ \Rightarrow\ & \frac{u'(c_t)}{u'(c_{t+1})} = \beta R + \frac{\lambda_t}{u'(c_{t+1})} \\ \Rightarrow\ & \frac{u'(c_t)}{u'(c_{t+1})} \geqslant \beta R > 1 \\ \Rightarrow\ & \frac{u'(c_{t+1})}{u'(c_t)} \leqslant \frac{1}{\beta R} < 1. \end{aligned}$$

[21] The multipliers associated with constraints $p_t \geqslant 0$, that is, ϕ_t, have already dropped out as we have started with the assertion that $p_t > 0$ for all $t \geqslant 2$.

This implies that the sequence $\{u'(.)\}$ is a contraction and converges to 0. By the Inada condition consumption converges to ∞. However, as the maximum possible consumption for any generation is bounded (by $(R+1)wh(e^{CM})$)[22], we have a contradiction.

Case 2(b): $p_1 = 0$. As $p_1 = 0, c_1 = c^{IM}$. Therefore, the period for which consumption rises above c^{IM} must be after 1. Let that period be k. As pension is positive for all $t > 1$, consumption rises and stays above c^{IM} for all subsequent periods. The proof is then the same as the proof of Case 2(a) above.

As we get a contradiction in all the cases, pension cannot remain positive throughout. Hence there exists some period, after the first, where pension becomes zero. □

Appendix E. Proof of Proposition 3

E.1. Proof of Proposition 3(a)

Proof. Suppose not, that is, suppose that $e_t < e^{CM} \quad \forall t$. By Lemma 3 the resource constraint binds in all periods. This implies that, $c_t = R(wh(e_{t-2}) - e_{t-1} - p_{t-1}) + p_t = p_t$ for all $t \geqslant 2$. On the other hand, the Pareto constraint requires that, in any $t, c_t \geqslant c^{IM}$. It follows that for all $t \geqslant 2, p_t = c_t \geqslant c^{IM} > 0$, that is, pension in every period (after the first) is strictly positive. But this contradicts Lemma 4. □

E.2. Proof of Proposition 3(b)

Proof. In the proof of Proposition 4 in Appendix F below we establish that $p_{t-1} \leqslant p_t$ for all $2 \leqslant t \leqslant T^*$ (that is, $p_1 \leqslant p_2 \leqslant \ldots \leqslant p_{T^*}$).[23] Since the resource constraint binds in all $t < T^*$, it follows from $p_{t-1} \leqslant p_t$ that $wh(e_{t-2}) - e_{t-1} \leqslant wh(e_{t-1}) - e_t$. For $t = T^*$, we have $wh(e_{T^*-2}) - e_{T^*-1} = p_{T^*-1} \leqslant p_{T^*} \leqslant wh(e_{T^*-1}) - e_{T^*}$. Shifting terms, we get $e_{T^*} - e_{T^*-1} \leqslant wh(e_{T^*-1}) - wh(e_{T^*-2})$. Since $e_{T^*-1} < e^{CM} = e_{T^*}$, it follows that $wh(e_{T^*-1}) - wh(e_{T^*-2}) > 0$ implying that $e_{T^*-2} < e_{T^*-1}$. Applying this argument recursively, we get that education is strictly rising till period T^* (that is, $e_1 < e_2 < \ldots < e_T^*$). □

Appendix F. Proof of Proposition 4

Proof. As T^* is the first such period when $e_t \geqslant e^{CM}$, education is strictly less than e^{CM} for all the previous periods and, by Lemma 3, the resource constraints bind in all these previous periods. By a similar argument as in the proof of Proposition 3(a), we have pensions to be strictly positive in all $2 \leqslant t \leqslant T^*$. In particular, pension is strictly positive in period 2. Now applying Lemma 2, we can argue inductively that pensions rise from period 2 till period T^*. Also, note that the Pareto constraint for period $2, c_2 \geqslant c^{IM}$, requires that $p_2 \geqslant R(e_1 + p_1)$, implying $p_2 \geqslant p_1$. Hence, pension rises from period 1 till T^*. □

Appendix G. Proof of Proposition 5

G.1. Proof of Proposition 5(a)

Proof. From Lemma 4 we know that there exists a time period $Z > 1$ such that $p_Z = 0$. From Proposition 4 we know that pensions increase till period T^*. Let $T^* + S$ be the period where pensions become zero for the first time. In Step 1 below we first establish the path of strictly falling pensions assuming that the resource constraints do not bind for periods $T^* < t < T^* + S$. Then in Step 2 we show that the path of strictly falling pensions is indeed consistent with non-binding resource constraints.

Step 1: *If the resource constraints do not bind for periods $T^* < t \leqslant T^* + S - 1$, then pensions fall strictly between periods $T^* + 1$ and $T^* + S$.*

Proof.

Since $p_{T^*+k}, p_{T^*+k+1}, p_{T^*+k+2} > 0$ for all $0 \leqslant k \leqslant S-3$, and the resource constraints do not bind, by Lemma 3 we know that $e_{T^*+k} = e^{CM} = e_{T^*+k+1}$ and $e_{T^*+k+2} \geqslant e^{CM}$. Moreover, $p_{T^*+k+2} > 0$ implies that $c_{T^*+k+2} \leqslant c_{T^*+k+3}$ by Lemma 1. Substituting the expressions for consumption in terms of education and pension we get

$$\begin{aligned}
& c_{T^*+k+2} \leqslant c_{T^*+k+3} \\
\Rightarrow\ & R(wh(e_{T^*+k}) - e_{T^*+k+1} - p_{T^*+k+1}) \\
& + p_{T^*+k+2} \leqslant R(wh(e_{T^*+k+1}) - e_{T^*+k+2} - p_{T^*+k+2}) + p_{T^*+k+3} \\
\Rightarrow\ & R(p_{T^*+k+2} - p_{T^*+k+1}) \leqslant p_{T^*+k+3} - p_{T^*+k+2} \\
& - R(e_{T^*+k+2} - e^{CM}) \\
& \qquad\qquad [\text{since } e_{T^*+k} = e^{CM} = e_{T^*+k+1}] \\
\Rightarrow\ & R(p_{T^*+k+2} - p_{T^*+k+1}) \\
& \leqslant p_{T^*+k+3} - p_{T^*+k+2}. \qquad [\text{since } e_{T^*+k+2} \geqslant e^{CM}]
\end{aligned}$$

Suppose $p_{T^*+k+2} \geqslant p_{T^*+k+1}$. This implies that $p_{T^*+k+3} \geqslant p_{T^*+k+2}$, and hence $p_{T^*+k+3} > 0$. By a recursive argument, S gets pushed to infinity and the pensions never become zero. This is a contradiction to Lemma 4. Hence $p_{T^*+k+2} < p_{T^*+k+1}$ for all $0 \leqslant k \leqslant S-3$, that is, pensions fall strictly between periods $T^* + 1$ and $T^* + S - 1$. Since $p_{T^*+S-1} > 0$ and $p_{T^*+S} = 0$, it follows that pensions fall strictly between periods $T^* + 1$ and $T^* + S$. □

Step 2: *The path of strictly falling pensions derived in Step 1 is consistent with the non-binding resource constraints.*

Proof.

We know that the resource constraint does not bind in period T^*, that is, $e_{T^*} + p_{T^*} < wh(e_{T^*-1})$. We first show that the resource constraint does not bind for period $T^* + 1$.

Since $p_{T^*+1} > 0$, by Lemma 1 we have $c_{T^*+2} \geqslant c_{T^*+1}$. It follows that

$$\begin{aligned}
& c_{T^*+2} \geqslant c_{T^*+1} \\
\Rightarrow\ & R(wh(e_{T^*}) - e_{T^*+1} - p_{T^*+1}) + p_{T^*+2} \\
& \geqslant R(wh(e_{T^*-1}) - e_{T^*} - p_{T^*}) + p_{T^*+1} \\
\Rightarrow\ & R(wh(e_{T^*}) - e_{T^*+1} - p_{T^*+1}) \geqslant R(wh(e_{T^*-1}) - e_{T^*} - p_{T^*}) \\
& + (p_{T^*+1} - p_{T^*+2}) \\
\Rightarrow\ & R(wh(e_{T^*}) - e_{T^*+1} - p_{T^*+1}) \\
& > R(wh(e_{T^*-1}) - e_{T^*} - p_{T^*}) \qquad [\text{since } p_{T^*+1} > p_{T^*+2} \text{ byStep 1}] \\
\Rightarrow\ & wh(e_{T^*}) - e_{T^*+1} - p_{T^*+1} > 0, \\
& \qquad [\text{since the resource constraint does not bind in period } T^*]
\end{aligned}$$

that is, the resource constraint does not bind for period $T^* + 1$.

[22] In presence of positive pensions, Lemma 3 implies that the maximum possible education for any generation is e^{CM} irrespective of whether the resource constraint binds. Pension allows consumption to be transferred to the previous generation. Hence, the maximum possible consumption level of a generation is the earnings it and its next generation generate, giving the upper bound $(Rwh(e^{CM}) + wh(e^{CM}) = (R+1)wh(e^{CM}))$.

[23] Note that the argument for rise of pensions relies only on Lemmas 2 and 3 and not on the rise of education subsidies, that is, there is no circularity in our argument.

Proceeding recursively as above, using in each step the non-binding resource constraint of the earlier period and strictly falling pensions between two consecutive periods, it is easy to see that the resource constraints do not bind for periods T^*+1 to T^*+S-1. Thus the path of strictly falling pensions is indeed consistent with the non-binding resource constraints assumed in Step 1. □

Combining Steps 1 and 2 we conclude that pensions fall strictly between periods T^*+1 and T^*+S. Also, since the resource constraints do not bind for periods T^* to T^*+S-1 while the pensions are strictly positive, it follows from Lemma 3 that education remains constant at e^{CM} from T^* to T^*+S-2. This completes the proof of Proposition 5(a). □

G.2. Proof of Proposition 5(b)

Proof. Similar to the proof of Proposition 5(a), we proceed in two steps. In Step 1, we prove that pension program is completely phased out from period T^*+S onwards by assuming that the resource constraint does not bind from period T^*+S onwards. Then in Step 2, we verify that zero pensions are consistent with non-binding resource constraints during this period.

Step 1: *If the resource constraints do not bind for periods $t \geqslant T^*+S$, then $p_t = 0\ \forall t \geqslant T^*+S$.*

Proof.

We know that $p_{T^*+S}=0$. Suppose that $p_{T^*+S+1}>0$. Since the resource constraint does not bind for period T^*+S, this implies that $e_{T^*+S}=e^{CM}$ (by Lemma 3) and $c_{T^*+S+2} \geqslant c_{T^*+S+1}$ (by Lemma 1). Substituting the expressions for consumption, we get

$$\begin{aligned}
& c_{T^*+S+2} \geqslant c_{T^*+S+1} \\
\Rightarrow\ & R(wh(e_{T^*+S}) - e_{T^*+S+1} - p_{T^*+S+1}) + p_{T^*+S+2} \\
& \geqslant R(wh(e_{T^*+S-1}) - e_{T^*+S} - p_{T^*+S}) + p_{T^*+S+1} \\
\Rightarrow\ & p_{T^*+S+2} - p_{T^*+S+1} \geqslant R(e_{T^*+S+1} - e^{CM} + wh(e_{T^*+S-1}) \\
& - wh(e^{CM}) + p_{T^*+S+1}). \\
& \qquad [\text{since } e_{T^*+S}=e^{CM},\ \text{and } p_{T^*+S}=0]
\end{aligned}$$

The RHS is strictly positive because $e_{T^*+S-1}, e_{T^*+S+1} \geqslant e^{CM}$ (by Lemma 3), and $p_{T^*+S+1}>0$. Hence pensions increase between periods T^*+S+1 and T^*+S+2. By a recursive argument pensions increase forever and never becomes zero, which is a contradiction to Lemma 4. This implies that $p_{T^*+S+1}=0$. By a similar argument $p_t=0$ $\forall t \geqslant T^*+S$. □

Step 2: *The path of zero pensions derived in Step 1 is consistent with the non-binding resource constraints for periods $t \geqslant T^*+S$.*

Proof.

Since by Step 1 pensions are zero from period T^*+S onwards, we have, for $t \geqslant T^*+S+1, c_t = R(wh(e_{t-2})-e_{t-1})$ which is strictly positive throughout as the Pareto constraint guarantees that $c_t \geqslant c^{IM} > 0$. It follows that the resource constraints do not bind for $t \geqslant T^*+S$. □

Combining Steps 1 and 2, the proof of Proposition 5(b) is completed. □

Appendix H. Proof of Proposition 6

Proof. We proceed in four steps. In the first two steps, we proceed under the assumption that the Pareto constraint does not bind from period T^*+S+1 onwards. In the third step, we show that the resulting solution path is consistent with this assumption. Finally, in the fourth step we show that education approaches the Golden Rule level in the limit.

Step 1: *If the Pareto constraint does not bind for periods $t \geqslant T^*+S+1$, then $\beta wh'(e_t) > 1\ \forall t \geqslant T^*+S-1$.*

Proof.

Suppose that $\beta wh'(e_{T^*+S-1}) \leqslant 1$. We know from the proof of Proposition 5 above that the resource constraint stops binding after period T^*. Consider the first order condition (7) for $t=T^*+S-1$ (using $\lambda_{T^*+S-1}, \lambda_{T^*+S} = \eta_{T^*+S+1} = \psi_{T^*+S-1} = 0$ due to complementary slackness):

$$\begin{aligned}
& -\beta R[u'(c_{T^*+S}) + \eta_{T^*+S}] + \beta^2 Rwh'(e_{T^*+S-1})[u'(c_{T^*+S+1})] = 0 \\
\Rightarrow\quad & u'(c_{T^*+S}) + \eta_{T^*+S} = \beta wh'(e_{T^*+S-1})(u'(c_{T^*+S+1})) \\
\Rightarrow\quad & u'(c_{T^*+S}) \leqslant \beta wh'(e_{T^*+S-1})(u'(c_{T^*+S+1})) \qquad [\text{since } \eta_{T^*+S} \geqslant 0] \\
\Rightarrow\quad & c_{T^*+S} \geqslant c_{T^*+S+1}. \qquad [\text{since } \beta wh'(e_{T^*+S-1}) \leqslant 1,\ \text{and } u''(.) < 0]
\end{aligned}$$

Substituting the expressions for consumption we get

$$\begin{aligned}
& c_{T^*+S} \geqslant c_{T^*+S+1} \\
& \Rightarrow R(wh(e_{T^*+S-2}) - e_{T^*+S-1} - p_{T^*+S-1}) + p_{T^*+S} \\
& \geqslant R(wh(e_{T^*+S-1}) - e_{T^*+S} - p_{T^*+S}) + p_{T^*+S+1} \\
& \Rightarrow e_{T^*+S} - e_{T^*+S-1} \geqslant [wh(e_{T^*+S-1}) - wh(e_{T^*+S-2})] \\
& \quad + p_{T^*+S-1} \qquad [\text{since } p_{T^*+S}, p_{T^*+S+1} = 0] \\
& \Rightarrow e_{T^*+S} - e_{T^*+S-1} > 0. \\
& \qquad [\text{since } e_{T^*+S-1} \geqslant e^{GR} > e^{CM} = e_{T^*+S-2},\ \text{and } p_{T^*+S-1} > 0]
\end{aligned}$$

Thus education strictly increases between T^*+S-1 and T^*+S which in turn implies that $\beta wh'(e_{T^*+S}) < 1$. By a recursive argument, it can be shown that education increases forever and consumption falls forever.

Now, consider a (feasible) deviation from this path where $e_t = e_{t+1} = e_{T^*+S-1}$ $\forall t \geqslant T^*+S-1$. It is easy to see that this deviation strictly dominates the original path as consumption increases between T^*+S and T^*+S+1 stays constant at a higher level from T^*+S+1 onwards, instead of falling. Hence the original path cannot be optimal, implying that $\beta wh'(e_{T^*+S-1}) > 1$. By a similar argument it can be shown that $\beta wh'(e_t) > 1$ $\forall t \geqslant T^*+S-1$. □

Step 2: *If the Pareto constraint does not bind for periods $t \geqslant T^*+S+1$, then $c_{t+1} > c_t$, and $e_t > e_{t-1}$ $\forall t \geqslant T^*+S$.*

Proof.

Under the assumption of non-binding Pareto constraint (the resource constraint is non-binding, as proved above), the first order condition (7) becomes

$$u'(c_{t+1}) = \beta wh'(e_t)u'(c_{t+2}) \qquad \forall t \geqslant T^*+S.$$

Then $c_{t+1} > c_t$ follows from this revised first order condition and Step 1.

Expanding $c_{T^*+S+2} > c_{T^*+S+1}$ (note that pensions are zero from period T^*+S onwards) we get

$$wh(e_{T^*+S}) - e_{T^*+S+1} > wh(e_{T^*+S-1}) - e_{T^*+S}$$

$$\Rightarrow wh(e_{T^*+S}) - wh(e_{T^*+S-1}) > e_{T^*+S+1} - e_{T^*+S}.$$

Suppose education falls, that is, the LHS is weakly negative. This implies that the RHS is strictly negative. By a recursive argument, education keeps on falling. Moreover, concavity of education production function along with $wh'(.) > 1$ implies that the rate of fall is increasing. This contradicts Lemma 3 which ensures that education stays above e^{CM} when the resource constraint does not bind.

Therefore we must have $e_{T^*+S} > e_{T^*+S-1}$. A similar argument shows that $e_t > e_{t-1}\ \forall t \geqslant T^* + S$. □

Step 3: *The Pareto constraint does not bind for periods $t \geqslant T^* + S + 1$ along the path derived in Steps 1 and 2.*

Proof.

From Step 2 we know that consumption increases strictly from period $T^* + S$. Hence, the Pareto constraint does not bind from $T^* + S + 1$. □

Step 4: *Education approaches the Golden Rule level in the limit, that is,* $\lim_{t\to\infty} e_t = e^{GR}$.

Proof.

We know that the Pareto constraint and the resource constraint stop binding after period $T^* + S + 1$. Then the first order condition (7) becomes

$$\frac{u'(c_{t+1})}{u'(c_{t+2})} = \beta w h'(e_t) \qquad \forall t \geqslant T^* + S + 1.$$

The sequence of e_t is monotonically increasing and bounded, hence convergent. By continuity, the sequences of consumption and marginal utilities are also convergent. The left hand side of the equation above converges to 1. Therefore, $\beta w h'(e_t)$ approaches 1, that is, e_t converges to e^{GR}.[24] □

Combining Steps 1, 2, 3 and 4, the proof of Proposition 6 is completed. □

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[24] Note that, with convergent consumption and non-binding resource and Pareto constraints, first order condition (8) implies zero pensions, hence verifying our result.